

A13483W1

SECOND PUBLIC EXAMINATION

Honour School of Physics Part B: 3 and 4 Year Courses

Honour School of Physics and Philosophy Part B

B4: Sub-Atomic Physics

TRINITY TERM 2015

Friday, 19 June, 2.30 pm – 4.30 pm

10 minutes reading time

Answer two questions.

Start the answer to each question in a fresh book.

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

1. (a) The rate of alpha decay of a nucleus can be expressed in terms of the probability of finding an alpha particle in the nucleus, the frequency of attempts for an alpha particle to escape, and the tunneling probability for the alpha particle to escape. Calculate the tunneling probability for a one-dimensional Hamiltonian

$$t_{1/2} = \frac{1}{\lambda} = \frac{1}{\nu P}$$

$$H = p^2/2m + V$$

where all symbols have their usual meaning, $V = 2Z\alpha_{EM}/r$, α_{EM} is the fine structure constant, and r is the distance from the centre of the nucleus. Express your answer in terms of the energy released in the decay (Q), the radius of the nucleus (r_a) and the atomic number of the daughter nucleus (Z).

(b) The alpha decay of $^{244}_{98}\text{Cf}$ has a Q -value of 7.33 MeV and a half-life of 19.4 minutes. Calculate the half-life of $^{224}_{90}\text{Th}$ given its Q -value of 5.52 MeV and assuming it has the same probability and frequency of finding alpha particles in the nucleus as $^{244}_{98}\text{Cf}$. Assume the nuclear radius of $^{244}_{98}\text{Cf}$ is 9.44 fm and the nuclear radius of $^{224}_{90}\text{Th}$ is 9.27 fm.

(c) Calculate the binding energy B of the alpha decay product of $^{224}_{90}\text{Th}$ using the formula

$$B = \alpha A - \beta A^{2/3} - \gamma \frac{(A - 2Z)^2}{A} - \epsilon \frac{Z^2}{A^{1/3}} + \delta$$

where A is the mass number, and the parameters $\alpha = 16$, $\beta = 18$, $\gamma = 23$, $\epsilon = 0.7$, and $\delta = 11/\sqrt{A}$ (in units of MeV). State what each term represents. Assuming the alpha decay dominates, is the $^{224}_{90}\text{Th}$ decay product likely to have a shorter or longer half-life than $^{244}_{98}\text{Th}$?

2. (a) For each of the following decays, state the interaction by which it proceeds, and give an example quark-flow or tree-level Feynman diagram (or say why the decay is not allowed):

(i) $p \rightarrow \pi^+ + \pi^0 + \pi^0$

(ii) $n \rightarrow p + e^- + \bar{\nu}_e$

(iii) $\pi^0 \rightarrow \gamma + \gamma$

(iv) $K^0 \rightarrow \pi^+ + \pi^-$

(v) $D^+ \rightarrow K^0 + \pi^+$

(vi) $\Delta^+ \rightarrow p + \pi^0$

(b) State which decay you expect to have the higher branching ratio, $\Xi^- \rightarrow \Lambda + \pi^-$ or $\Xi^- \rightarrow n + \pi^-$, and why. The Ξ^- baryon has strangeness quantum number -2 .

(c) The J/ψ meson can decay via a virtual photon. If this were the only possible decay mode, what would be the branching fraction for $J/\psi \rightarrow \mu^+ + \mu^-$?

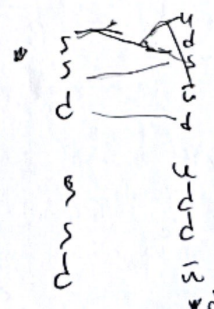
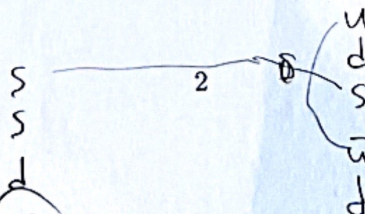
(d) The J/ψ meson can also decay via a three-gluon intermediate state. Using the observed branching ratio of 6% for $J/\psi \rightarrow \mu^+ + \mu^-$, estimate the value of the strong coupling constant α_s and state any assumptions you make.

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24%

76%

ssd



3. (a) Assume a nucleus of charge Z and radius a has a uniform charge distribution. Calculate the electromagnetic form factor $F(\Delta k)$ associated with this nucleus, where Δk is the change in momentum of a projectile. [10]

(b) Determine the differential cross section $d\sigma/d\Omega$ for an electron scattering off this nucleus, using the Born approximation and the Rutherford cross section, [4]

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} = \frac{Z^2 \alpha_{\text{EM}}^2}{4 E^2 \sin^4(\theta/2)},$$

where α_{EM} is the fine structure constant, E is the energy of the incident electron, and θ is the scattering angle. [6]

(c) An electron with energy 1 GeV scatters off a proton at rest in the laboratory. Calculate the differential cross section $d\sigma/d\Omega$ for an observed electron with energy 800 MeV. Assume that the proton charge distribution is uniform within a radius of 1 fm. [5]

(d) How realistic is the assumption of a uniform charge distribution for a proton at high scattering energies? What interactions within the proton affect the charge distribution as the scattering energy increases? At very high energies what particles within the proton do the electrons scatter off? [5]

$$\frac{\alpha_{\text{EM}}^2}{4\alpha_{\text{EM}}^2 + 9\alpha_s^4} = \frac{6}{9}$$

$$q_{\text{ES}} (1 - 0.06 \times 4) \alpha_{\text{EM}}^2 = 9 \alpha_s^4$$

4. (a) The W boson mass has been measured to high precision at electron-positron and proton-antiproton colliders using the processes $e^+ + e^- \rightarrow W^+ + W^-$ and $p + \bar{p} \rightarrow W + X$, respectively. Draw two lowest-order Feynman diagrams for the $e^+ + e^- \rightarrow W^+ + W^-$ process and one for the $p + \bar{p} \rightarrow W + X$ process. Why is it not possible to produce a single W boson with no additional particle in an electron-positron collider?

[5]

(b) Write down the decay modes of the W boson. What are the best decay modes for observing the W boson at a proton-antiproton collider? What is the branching fraction for each of these best decay modes?

[5]

(c) The cross section for W boson production at a proton-antiproton collider with a centre-of-mass energy of 1.96 TeV is 25 nb. The total cross section for inelastic scattering is 30 mb, and 3×10^{14} inelastic events were produced in each detector by the Tevatron. How many W boson events were produced in each detector? How many W bosons were produced in the best decay modes for observation in each detector [from (b)]?

[3]

(d) In proton-antiproton collisions with a centre-of-mass energy of 2 TeV, a W boson is produced by constituents of the proton and the antiproton. If these interacting particles have a fraction x_p of the proton's energy and $x_{\bar{p}}$ of the anti-proton's energy, what is the value of the product $x_p x_{\bar{p}}$ if they produce a single W boson with mass equal to 80 GeV? In the case where the fractions are equal, what is $x = x_p = x_{\bar{p}}$?

[3]

(e) The mass of the W boson produced can be reconstructed as the centre-of-mass energy of the final-state $l\nu$ system. Writing the four-vectors of the final-state particles as $(E^l, p_x^l, p_y^l, p_z^l)$ and $(E^\nu, p_x^\nu, p_y^\nu, p_z^\nu)$, write down the square of the centre-of-mass energy of this system (neglecting lepton mass) and use it to obtain the expression for m , the invariant mass of the system.

[3]

(f) How is the neutrino momentum measured in a collider experiment? Why is it not possible to measure the neutrino momentum along the beam direction in a proton-antiproton collider?

[3]

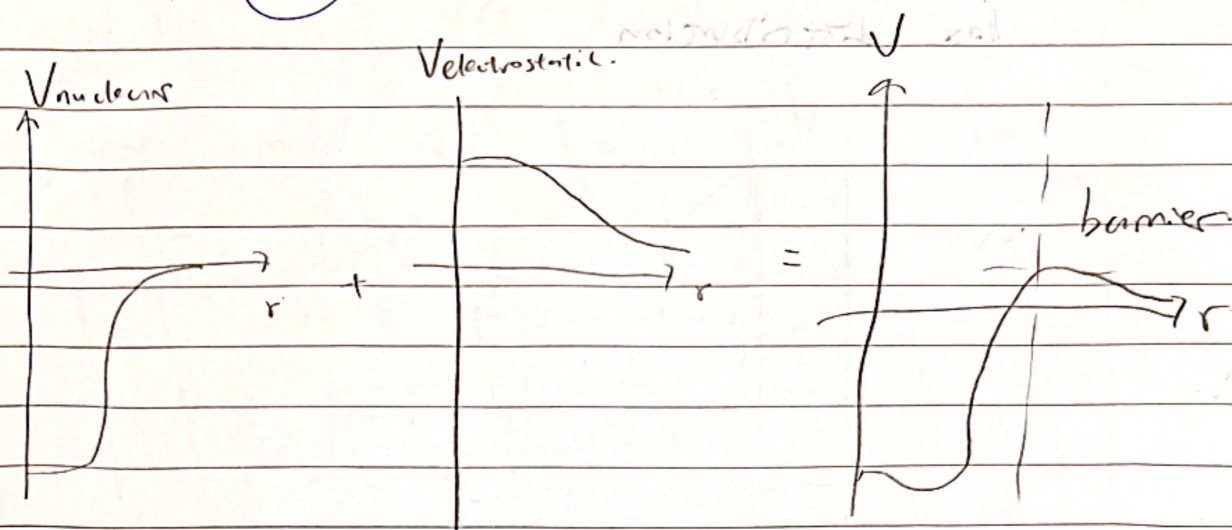
(g) In the expression for the invariant mass, set p_z^l and p_z^ν to zero to obtain an expression for the transverse mass m_T .

[2]

(h) In the case where a charged lepton and neutrino have opposite non-zero p_z , is the transverse mass larger or smaller than the invariant mass?

[1]

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$$V = \frac{2Ze^2}{r}$$

1D potential well (~~tunneling~~) (tunneling)

- Amplitude of the wave function drops exponentially

$$\propto e^{-kx} \quad (k = \sqrt{2m(V_0 - E)/\hbar^2})$$

V_0 : potential

E : energy of particle

- so for wall thickness d , ~~tunneling~~ tunneling probability $\propto e^{-2kd}$

in ~~nucleus~~ nucleus, $V = V(r)$

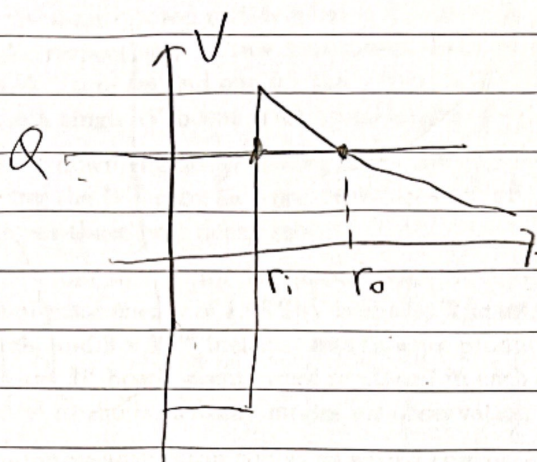
$$\therefore P_{\text{tunneling}} = P = e^{-2 \int k(r) dr} = e^{-2G}$$

WKB Approximation

$$G = \int k(r) dr = \int \sqrt{\frac{2m}{\hbar^2}} \sqrt{V - E} dr$$

$$= \sqrt{\frac{2mE}{\hbar^2}} \int \sqrt{\frac{2Ze^2}{Qr} - 1} dr$$

replace saxon-woods distribution with a box distribution



~~$r_i = r_0 A^{1/3}$~~ r_i = size of nucleus ($\propto A^{1/3}$)

r_0 = radius where kinetic exceeds potential energy $\rightarrow$ free particle.

$$(\text{at } r_0) V(r_0) = Q \therefore r_0 = \frac{2Ze^2m}{Q}$$

$$G = \sqrt{\frac{2mE}{\hbar^2}} \int_{r_i}^{r_0} \sqrt{\frac{r_0}{r} - 1} dr$$

$$r = r_0 \cos^2 \theta \quad dr = -2r_0 \cos \theta \sin \theta d\theta$$

$$G = \sqrt{\frac{2mE}{\hbar^2}} 2r_0 \int_0^{\theta_i} \sqrt{\frac{1}{\cos^2 \theta} - 1} \cos \theta \sin \theta d\theta \quad \left| \theta_i = \arccos\left(\sqrt{\frac{r_i}{r_0}}\right) \right.$$

$$= \sqrt{\frac{2mE}{\hbar^2}} r_0 \int_0^{\theta_i} 2 \sin^2 \theta d\theta$$

$$= \sqrt{\frac{2mE}{\hbar^2}} r_0 [\theta - \cos \theta \sin \theta]_0^{\theta_i}$$

$$= \sqrt{\frac{2mQ}{\hbar^2}} r_0 \left[\alpha \cos^{-1} \sqrt{\frac{r_i}{r_0}} - \sqrt{\frac{r_i}{r_0}} \sqrt{1 - \frac{r_i}{r_0}} \right]$$

for small Q , terms in $\left[\dots \right] \sim \frac{\pi}{2}$

$$\therefore G = \sqrt{\frac{2mQ}{\hbar^2}} r_0 \frac{\pi}{2}$$

$$= \sqrt{\frac{2mQ}{\hbar^2}} \frac{2Z\alpha_{EM}}{Q} \frac{\pi}{2}$$

$$= \left(\alpha_{EM} \pi \sqrt{\frac{2m}{\hbar^2}} \right) \frac{Z}{\sqrt{Q}}$$

$\therefore$ tunneling prob. probability

$$P = \exp(-2G) \quad p = e^{-2G}$$

$$P = \exp \left[-2 \alpha_{EM} \pi \sqrt{\frac{2m}{\hbar^2}} \frac{Z}{\sqrt{Q}} \right]$$

$$r_i = r_a \quad p = e^{-2G} \quad r_0 = \frac{2Z\alpha_{EM}}{Q}$$

$$\therefore \left[P = \exp \left[-4 \alpha_{EM} \sqrt{\frac{2m}{\hbar^2}} \left(\frac{Z}{\sqrt{Q}} \right) \left[\cos^{-1} \sqrt{\frac{r_a}{r_0}} - \sqrt{\frac{r_a}{r_0}} \sqrt{1 - \frac{r_a}{r_0}} \right] \right] \right]$$

$$\left(r_0 \equiv \frac{2Z\alpha_{EM}}{Q} \right)$$

(b) substance numbers

$$\alpha_{EM} = \frac{1}{137} \quad r_0 = 2\alpha_{EM} \frac{Z}{Q}$$

$$\alpha_{EM} = \frac{e^2}{4\pi\epsilon_0} = \frac{e^2}{4\pi\epsilon_0\hbar c}$$

(we've assumed $\hbar = c = 1$)

~~If we replace $\hbar, c$ by SI units,~~

~~$$\alpha_{EM} = \frac{1}{137} \frac{1}{1.05 \times 10^{-34} \times 3 \times 10^8} = 2.317 \times 10^{23}$$~~

$$\therefore \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137} \quad \frac{e^2}{4\pi\epsilon_0} = \frac{\hbar c}{137}$$

$$= 2.3 \times 10^{-28} \text{ J}\cdot\text{m}$$

$$= 2.3 \times 10^{-13} \text{ J}\cdot\text{fm}$$

$$= 1.44 \times 10^6 \text{ eV}\cdot\text{fm}$$

$$= 1.44 \text{ MeV}\cdot\text{fm} = \alpha_{EM}$$

$$\therefore r_0 = 2\alpha_{EM} \frac{Z}{Q} = 2 \cdot 1.44 \frac{Z}{Q}$$

$$= 2.88 \frac{Z}{Q} \text{ (fm)}$$

$$\text{Now } -4\alpha_{EM} \sqrt{\frac{2m}{\hbar^2}} = -4 \times 1.44 \text{ MeV}\cdot\text{fm} \times \sqrt{\frac{2m}{\hbar^2}}$$

$$[\hbar] = \text{J}\cdot\text{s} \quad [m] = \frac{\text{J}}{[\text{c}^2]} = \frac{\text{J}\cdot\text{s}^2}{\text{m}^2}$$

$$\therefore \left[\sqrt{\frac{2m}{\hbar^2}} \right] = \left(\frac{\text{J}\cdot\text{s}^2}{\text{m}^2 \cdot \text{J}^2 \cdot \text{s}^2} \right)^{\frac{1}{2}} = \text{m}^{-1} \text{J}^{-1/2}$$

~~SI units~~ $m = m(\alpha\text{-particle})$

~~$\sqrt{\frac{2m}{\hbar^2}}$~~

$$\therefore \left[\sqrt{\frac{2m}{\hbar^2}} \right] = \text{fm}^{-1} \text{MeV}^{-1/2}$$

$$M_{\alpha} = m_{\alpha\text{-particle}} = 2m_p + 2m_n$$

$$= 2(938.3) + 2(939.6) = 3755.8 \frac{\text{MeV}}{c^2}$$

$$\therefore \sqrt{\frac{2m}{\hbar^2}} = \sqrt{\frac{2 \times 3755.8 \text{ MeV}}{(\hbar c)^2}}$$

$$(\hbar c) = 137 \times 1.44 \text{ MeV} \cdot \text{fm} = 197.3 \text{ MeV} \cdot \text{fm}$$

$$\therefore \sqrt{\frac{2m}{\hbar^2}} = \left(\frac{2 \times 3755.8}{(197.3)^2} \right)^{1/2} \text{ MeV}^{-1/2} \text{ fm}^{-1}$$

$$= 0.44 \text{ MeV}^{-1/2} \text{ fm}^{-1}$$

$$\therefore -4\alpha_{\text{EM}} \sqrt{\frac{2m}{\hbar^2}} = -4 \cdot 1.44 \text{ MeV} \cdot \text{fm} \cdot 0.44 \text{ MeV}^{-1/2} \text{ fm}^{-1}$$

$$= -2.534 \text{ MeV}^{-1/2}$$

$$\therefore P = e^{-\left(2.534 \frac{Z}{A} \left[\cos^{-1} \sqrt{\frac{r_a}{2.88 \frac{Z}{A}}} - \sqrt{\frac{r_a}{2.88 \frac{Z}{A}}} \sqrt{1 - \frac{r_a}{2.88 \frac{Z}{A}}} \right] \right)}$$

For $^{244}_{98}\text{Cf}$, product $Z=98-2=96$.

$$Q = 87.33$$

$$\therefore r_0 = 2.88 \frac{Z}{Q} = 2.88 \frac{96}{7.33} = 37.7$$

$$\cos^{-1} \frac{r_a}{r_0} = 9.44$$

$$\therefore \frac{r_a}{r_0} = \frac{9.44}{37.7} = 0.25 \quad \sqrt{\frac{r_a}{r_0}} = 0.5$$

$$\cos^{-1}(0.25)$$

$$\cos^{-1}(0.5) - 0.5 \sqrt{1-0.25} = 0.614$$

$$P_{cf} = e^{-\left(2.534 \times 0.614 \times \frac{96}{\sqrt{7.33}}\right)}$$

$$= e^{-55.17}$$

For $^{224}_{90}\text{Th}$, product $Z=90-2=88$

$$Q = 5.52 \quad r_a = 9.27$$

$$r_0 = 2.88 \frac{88}{5.52} = 45.9$$

$$\frac{r_a}{r_0} = \frac{9.27}{45.9} = 0.2 \quad \sqrt{\frac{r_a}{r_0}} = 0.45$$

$$\cos^{-1}(0.45) - 0.45 \sqrt{1-0.2} = 0.70$$

$$\therefore P_{th} = 0.19 (-2.534 \times 0.70 \times \frac{88}{\sqrt{5.52}})$$

$$= e^{-66.44}$$

the half life $t_{1/2} = \ln 2 \times \text{decay time } \tau$

$$\propto \frac{1}{P}$$

$$\therefore t_{1/2} \propto \frac{1}{P}$$

$$t_{1/2}(cf) \propto \frac{1}{P_{cf}}$$

$$\Rightarrow t_{1/2}(Th) = \frac{P_{cf}}{P_{th}} t_{1/2cf}$$

$$t_{1/2}(Th) \propto \frac{1}{P_{th}}$$

$$\therefore t_{1/2}(Th) = (19.4 \text{ min}) \frac{e^{-55.12}}{e^{-66.44}}$$

$$= (19.4 \text{ min}) e^{11.27}$$

$$= 1.5 \times 10^6 \text{ min}$$

$$(2.12 \times 10^6 \text{ min})$$

- α decay product of ${}^{224}_{90}\text{Th}$ is has

$$A = 224 - 4 = \underline{220}, \quad Z = 90 - 2 = \underline{88}$$

$$B = \alpha A - \beta A^{2/3} - \gamma \frac{(A - 2Z)^2}{A} - \epsilon \frac{Z^2}{A^{1/3}} + \delta$$

αA : Volume term. the strongly attractive force

between nucleons acts between nearest neighbours. So each nucleon bind same number of nucleons. So this contribution scales with number of nucleons A

$-\beta A^{2/3}$: Surface term: The nucleons at the surface do not interact with same number of neighbours as the ones in the middle. So total binding energy is reduced by the amount proportional to number of nucleons at the surface ($A^{2/3}$)

$-\gamma \frac{(A - 2Z)^2}{A}$: Asymmetry term. This is the result of Pauli exclusion principle. Two protons or two neutrons cannot occupy the same energy state, but one proton and one neutron can. So difference in proton and neutron numbers will cause a reduction in binding energy (less stable)

$-\epsilon \frac{Z^2}{A^{1/3}}$: the ~~Coulomb~~ Coulomb repulsion between charged particles (protons), reduces the binding energy.

δ : pairing term. p-p and n-n pair tends to be more stable configurations so even number of p + even number of n is most stable. odd number of p and n is least stable.

α decay probability scales with $e^{-\frac{Z}{\sqrt{Q}}}$

For ${}^{224}_{90}\text{Th}$, $\frac{Z}{\sqrt{Q}} = \frac{90-2}{\sqrt{15.52}} = 37.5$.

~~For~~ For its product, $Z = 88 - 2 = \underline{86}$

$$Q = M(88, 220) - M(86, 216) - M(2, 4).$$

~~M(88)~~ $(M = M(Z, A))$ $(B = B(A, Z))$

$$M(88, 220) = 88m_p + (220 - 88)m_n - B(88, 220).$$

$$M(86, 216) = 86m_p + (216 - 86)m_n - B(86, 216)$$

$$M(2, 4) = 2m_p + (4 - 2)m_n - B(2, 4)$$

$$\therefore B(2, 4) = B(2, 4) + B(86, 216) - B(88, 220)$$

~~$$B(2, 4) = 16 \times 4 = 18 \times 4^{2/3} - 23 \times \frac{(4-4)^2}{4} = 0.7 \frac{2^2}{4^{1/3}}$$~~

~~$$+ \frac{11}{\sqrt{4}} = 22.38 \text{ MeV}$$~~

$$B(86, 216) = 16 \times 216 - 18 \times 216^{2/3} - 23 \times \frac{(216 - 2 \times 86)^2}{216} \\ - 0.7 \frac{86^2}{216^{1/3}} + \frac{11}{\sqrt{216}}$$

$$= ~~1739~~ 1739.73 \text{ MeV}$$

$$B(88, 220) = 16 \times 220 - 18 \times 220^{2/3} - 23 \times \frac{(220 - 2 \times 88)^2}{220} \\ - 0.7 \times \frac{88^2}{220^{1/3}} + \frac{11}{\sqrt{220}} \\ = 1764.41$$

$$B(90, 224) = 16 \times 224 - 18 \times 224^{2/3} - 23 \times \frac{(224 - 2 \times 90)^2}{224} \\ - 0.7 \times \frac{90^2}{224^{1/3}} + \frac{11}{\sqrt{224}} \\ = 1788.43$$

$$\therefore 1764.41 + B(2, 4) - 1788.43 = 5.52$$

$$B(12, 4) = 29.54$$

Q for product of ${}^7_6\text{Li} + {}^{12}_4\text{O}$

$$Q = 1739.73 + 29.54 - 1764.41 \\ = 4.86$$

The product $^{220}_{88}\text{Ra}$ has $\frac{Z}{A} = \frac{86}{220} = 39$

$$\therefore \frac{Z}{A} \text{ of } ^{220}_{88}\text{Ra} > \frac{Z}{A} \text{ of } ^{224}_{90}\text{Th}$$

$$\therefore P(\text{Ra}) < P(\text{Th})$$

$\therefore$ product ($^{220}_{88}\text{Ra}$) has longer
lifetime

$$77 = \frac{80}{\frac{100}{100}} = \frac{80}{100} \text{ is a } 80\% \text{ number}$$

$$17 \text{ is } \frac{17}{100} \text{ of } 100$$

$$17 \text{ is } 17\% \text{ of } 100$$

$$17 \text{ is } 17\% \text{ of } 100$$

$$17 \text{ is } 17\% \text{ of } 100$$

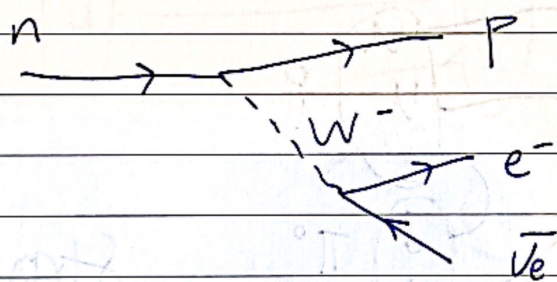
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(a)

(i) $p \rightarrow \pi^+ + \pi^0 + \pi^0$

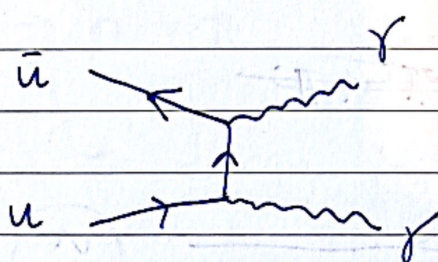
not possible. p is stable due to baryon number conservation (no lighter baryons than p)

(ii) $n \rightarrow p + e^- + \bar{\nu}_e$



Weak

(iii)

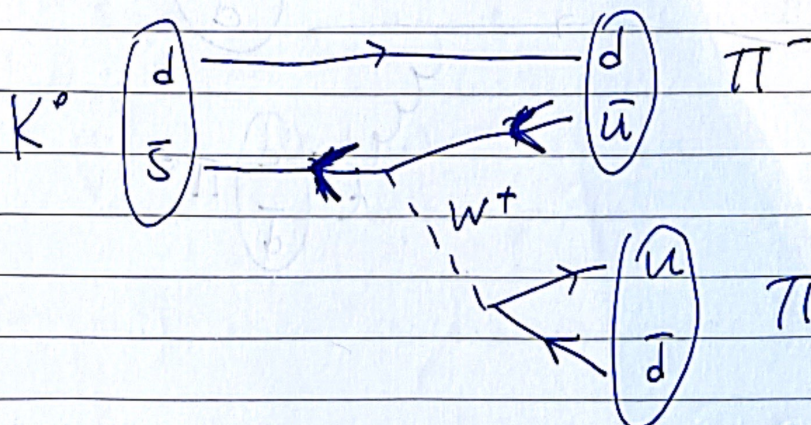


$\pi^0 \rightarrow \gamma + \gamma$

electromagnetic

(iv)

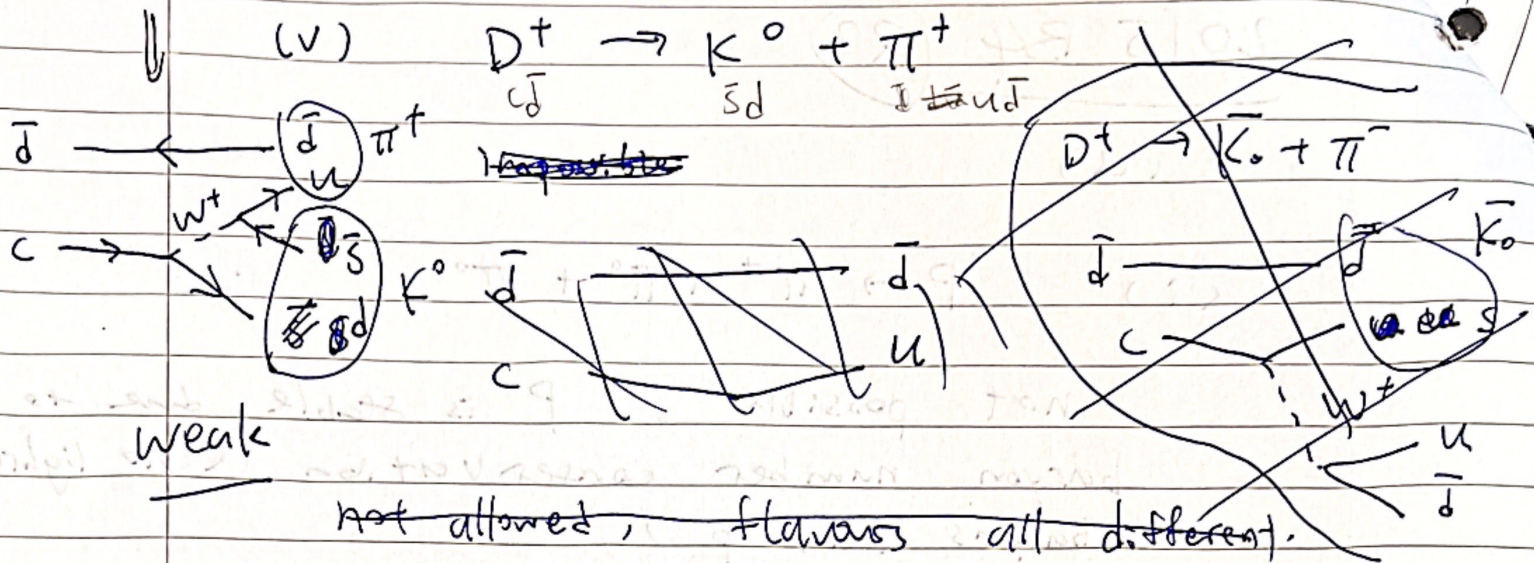
$K^0 \rightarrow \pi^+ + \pi^-$



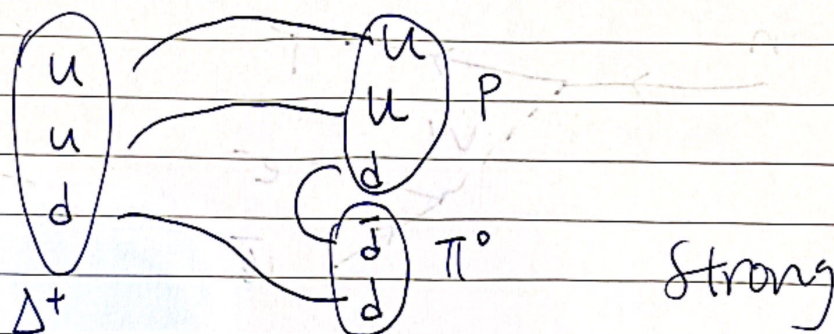
π^-

π^+ Weak

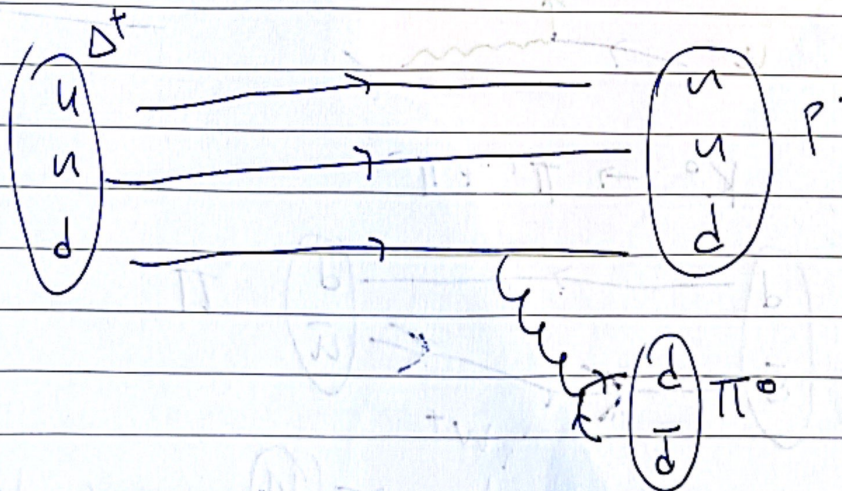
double
Cabbibo suppresses



(vi) $\Delta^+ \rightarrow p + \pi^0$



(v) $K^0 \rightarrow \pi^+ + \pi^-$



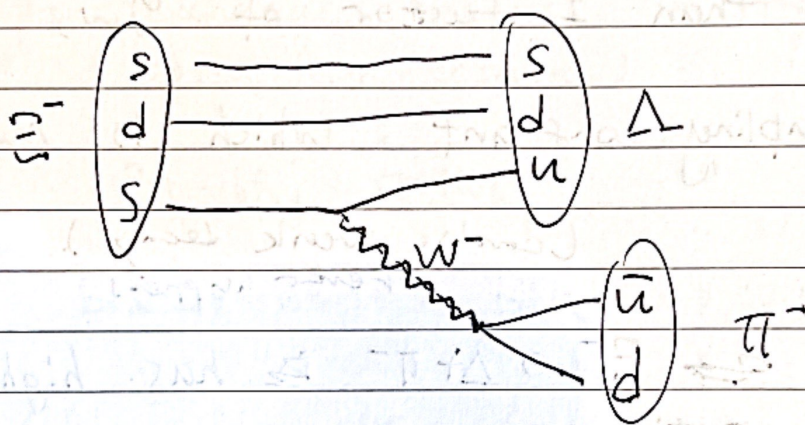
2(b) $\because \Xi^-$ baryon has strangeness -2 and charge -1

$\therefore$ the quark content has to be ~~uss~~ dss

Also, neutron (udd) $\pi^- (\bar{u}d)$ $\Delta (uds)$

$\therefore$ For $\Xi^- \rightarrow \Delta + \pi^-$ strangeness change $\Delta S = 1 \neq 0$
 $S = -2 \quad S = -1 \quad S = 0$

$\therefore$ only weak interaction possible



If $\Xi^- \rightarrow n + \pi^-$, then we want

~~the~~ dss to decay into udd and $\bar{u}d$

$\Delta S = 2 \neq 0$ only weak decay possible

Both S quarks need to change flavour

$\therefore$ 2 virtual particles are needed
exchange

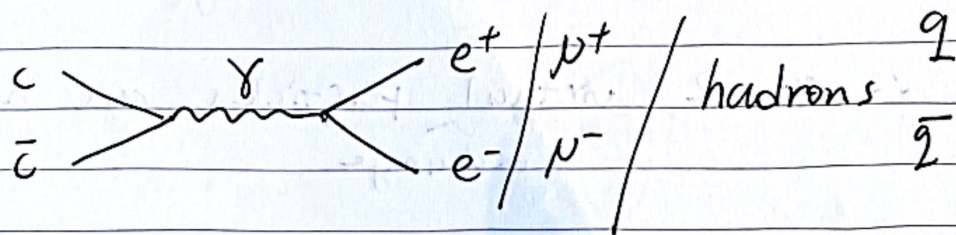
for this decay process, if this process is possible.

Thus $\Xi^- \rightarrow n + \pi^-$ will definitely have one more vertex than $\Xi^- \rightarrow \Lambda_0 + \pi^-$ and therefore even if we can draw a ~~Feynman~~ Feynman diagram for $\Xi^- \rightarrow n + \pi^-$, the cross section for this process will be suppressed by more than 1 factor of g_w , the weak coupling constant, which is much less than 1 (double weak decay never happens)

Hence ~~$\Xi^- \rightarrow \Lambda + \pi^-$~~ $\Xi^- \rightarrow \Lambda + \pi^-$ has higher branching ratio

(c) J/ψ is a state of charmonium ($c\bar{c}$) (the ground state)

The possible decay modes via ~~a~~ virtual photon is given by



The $\tau^+\tau^-$ pair is impossible because

Since the mass of J/ψ meson is $3.10 \text{ GeV}/c^2$

Mass of T particle is $1.77 \text{ GeV}/c^2$

$$\therefore \cancel{M_{J/\psi} > 2 \times M_T} \therefore M_{J/\psi} < 2 \times M_T$$

We have negative Q value, thus decay impossible.

→ All leptons ~~couple~~ couple to the ~~weak exchange~~ exchange bosons in the same way (lepton universality)

$\therefore$ Branching ratios

$$\text{Br}(J/\psi \rightarrow e\bar{e}) = \text{Br}(J/\psi \rightarrow \mu\bar{\mu})$$

→ Now for the $\text{Br}(J/\psi \rightarrow \text{hadrons})$:

Because the EM coupling constant

$$\alpha_{\text{EM}} = \frac{e^2}{(4\pi\epsilon_0)\hbar c} \propto e^2 \propto (\text{charge})^2$$

and cross-section of EM decay

$$\sigma = \frac{4\pi}{3} \hbar^2 c^2 \frac{\alpha_{\text{EM}}^2}{S}$$

S is CM frame energy and is the same for all 3 processes since they all ~~start~~ start with the J/ψ states

$$\therefore \sigma \propto \alpha_{\text{EM}}^2 \propto \alpha_{\text{EM}}(c\bar{c}) \times \alpha_{\text{EM}}(\text{products})$$

~~∴ $\sigma(\psi \rightarrow \mu\mu)$~~

$$\sigma(\psi \rightarrow \mu\mu) \propto (\text{charge of } c)^2 \times (\text{charge of } \mu)^2$$

$$\propto \left(\frac{2}{3}\right)^2 \times (1)^2$$

$$\sigma(\psi \rightarrow \text{hadrons}) \propto (\text{charge of } c)^2 \times$$

$$(\text{charge of product quarks})^2$$

$$\propto \left(\frac{2}{3}\right)^2 \times (\text{charge of product quarks})^2$$

$$\text{The ratio } R = \frac{\sigma(\psi \rightarrow \text{hadrons})}{\sigma(\psi \rightarrow \mu\mu)}$$

$$\therefore R = \frac{\sum_i q_i^2}{1} \times 3 \quad \text{index } i \text{ is the types of quarks can be produced by the decay of } \psi$$

$\therefore$ u, d, s quarks are lighter than $\frac{1}{2}$ of mass of ψ , b, t quarks are heavier

$$\therefore R = \frac{(q_u)^2 + (q_d)^2 + (q_s)^2}{1} \times 3$$

$$= \left[\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 \right] \times 3 = 2$$

where the $\times 3$ factor comes from the color degeneracy

$$\therefore \text{Br}(J/\psi \rightarrow \text{hadrons}) = R \times \text{Br}(J/\psi \rightarrow \mu\mu)$$

$$= 2 \times \text{Br}(J/\psi \rightarrow \mu\mu)$$

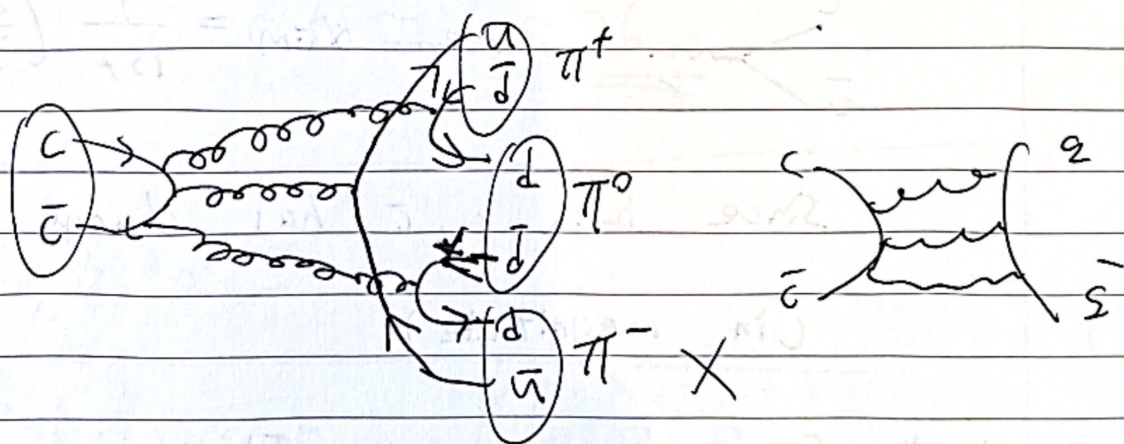
~~total~~

$$\therefore \text{Br}(J/\psi \rightarrow ee) = \text{Br}(J/\psi \rightarrow \mu\mu)$$

$$\therefore \text{Br}(J/\psi \rightarrow \mu\mu) = \frac{1}{2+1+1} \times 100\% = 25\%$$

(d) J/ψ decay via 3 gluons (1 $\rightarrow$ color singlet
2 $\rightarrow$ parity doesn't work)

one example is $J/\psi \rightarrow \pi^0 + \pi^+ + \pi^-$



$\therefore$ Both γ and gluon are massless bosons

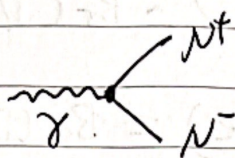
$\therefore$ their propagator factors are the same because

$$\text{in } \frac{1}{q^2 - m^2} \quad \text{both } m = 0$$

and q both equal to the centre of mass energy of J/ψ

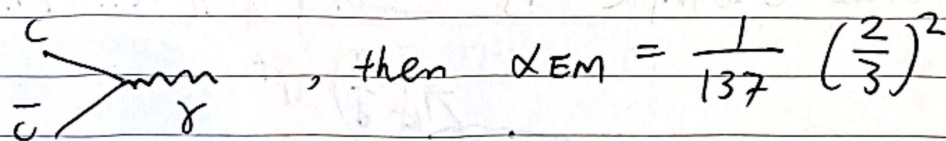
$\therefore$ The only contribution to the ratio between cross sections (thus branching ratios) is the coupling ~~for~~ constant

If a ~~wa vertex~~ EM vertex like



then $\alpha_{EM} = \frac{1}{137}$ $\because$ both N^+, N^- has charge 1 (magnitude)

If a ~~wa~~ EM vertex like :



, then $\alpha_{EM} = \frac{1}{137} \left(\frac{2}{3}\right)^2$

Since both $c, \bar{c}$ has charge $\frac{2}{3}$ (in magnitude)

And for the process $J/\psi \rightarrow ggg$ (3 gluons) there are 6 vertices in total

So contribution factor is α_s^6

Since $Br(J/\psi \rightarrow \mu\mu) = 6\%$

$\therefore Br(J/\psi \rightarrow \gamma\gamma) = 6\% \times 4 = 24\%$

Assumption: J/ψ can only ~~decay~~ decay via a virtual photon or 3 gluons

→ ~~Br(J/ψ)~~ $\text{Br}(J/\psi \rightarrow ggg) = 1 - 24\% = 76\%$

$$\therefore \frac{\text{Br}(J/\psi \rightarrow ggg \rightarrow)}{\text{Br}(J/\psi \rightarrow \mu\mu)} = \frac{\alpha_s^6}{\cancel{6} \left(\frac{1}{137}\right) \times \left(\frac{1}{137}\right) \left(\frac{2}{3}\right)^2}$$

$$= \frac{76}{6}$$

$$\Rightarrow \alpha_5 \approx \underline{\underline{0.26}}$$

$$g^2 g_p^2 \propto E_m^2$$

$\sum_i q_i^2 N_i^2 \alpha_{em}^2 + 2s^6$

$\downarrow$

3

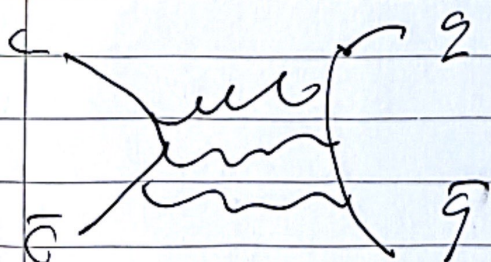
$\downarrow$

3.3

$\downarrow$

4 = f # colors

quark



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$$P(1) = 1 - 1 = 0 \quad (1 - 1) = 0$$

$$P(2) = 1 - 1 = 0 \quad (1 - 1) = 0$$

$$P(3) = 1 - 1 = 0$$

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$$P(4) = 1 - 1 = 0$$

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2015 B4 (Q3)

nuclear form factor $F(\Delta k)$

$$F(\Delta k) = \int e^{i \underline{r} \cdot \Delta \underline{k}} \rho(\underline{r}) d^3 \underline{r} \quad (\Delta \underline{k} = \underline{k} - \underline{k}')$$

$$= \int e^{i r \Delta k \cos \theta} \rho(r) r^2 \sin \theta dr d\theta d\phi$$

uniform charge distribution $\rho = \text{constant}$

$$\int_{\text{sphere } r=a} \rho d^3 \underline{r} = 1 \Rightarrow \rho \frac{4}{3} \pi a^3 = 1 \Rightarrow \rho = \frac{3}{4\pi a^3}$$

$$\therefore F(\Delta k) = \frac{3}{4\pi a^3} \int_{-1}^1 e^{i \Delta k r \cos \theta} \int_0^a r^2 dr d\cos \theta \int_0^{2\pi} d\phi$$

$$= \frac{3}{4\pi a^3} \int_0^a \frac{1}{\Delta k r} \left[e^{i \Delta k r} - e^{-i \Delta k r} \right] r^2 dr$$

$$= \frac{3}{4\pi a^3 \Delta k} \int_0^a \sin(\Delta k r) r dr$$

integration by parts :

$$\begin{aligned}
 & \int_0^a \sin(\Delta k r) r dr \\
 & = \int u dv \\
 & = uv - \int v du \\
 & = \frac{1}{2} r^2 \sin(\Delta k r) - \int
 \end{aligned}$$

$$\begin{aligned}
 u &= \sin(\Delta k r) \\
 du &= \Delta k \cos(\Delta k r) dr \\
 dv &= r dr \\
 v &= \frac{1}{2} r^2
 \end{aligned}$$

$$\begin{aligned}
 u &= r & du &= dr \\
 dv &= \sin(\Delta k r) dr \\
 v &= -\frac{1}{\Delta k} \cos(\Delta k r)
 \end{aligned}$$

$$\int \sin(\Delta k r) r dr = \int u dv = uv - \int v du$$

$$= -\frac{1}{\Delta k} r \cos(\Delta k r) + \int \frac{1}{\Delta k} \cos(\Delta k r) dr$$

$$= -\frac{1}{\Delta k} r \cos(\Delta k r) + \frac{1}{\Delta k^2} \sin(\Delta k r)$$

$$= \frac{1}{\Delta k^2} (\sin(\Delta k r) - r \cos(\Delta k r))$$

$$\therefore \int_0^a \sin(\Delta k r) r dr = \frac{1}{\Delta k^2} (\sin(\Delta k a) - a \cos(\Delta k a))$$

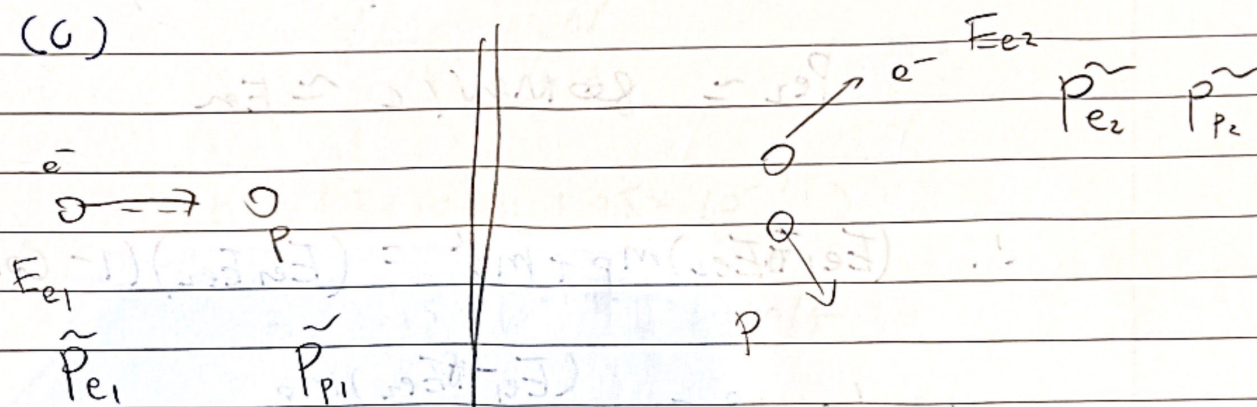
$$\therefore F(\Delta k) = \frac{3}{a^3 \Delta k} \frac{1}{\Delta k^2} (\sin(\Delta k a) - \Delta k a \cos(\Delta k a))$$

$$= \frac{3 (\sin(\Delta k a) - a \cos(\Delta k a))}{(a \Delta k)^3}$$

$$(b) \quad \frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{Rutherford}} |F(\Delta k)|^2$$

$$= \frac{Z^2 \alpha_{\text{EM}}^2}{4E^2 \sin^4(\frac{\theta}{2})} \left| \frac{3(\sin(\Delta k a) - \Delta k a \cos(\Delta k a))}{(\Delta k a)^3} \right|^2$$

(c)



4-vector conservation (4 momentum)

$$\vec{p}_{e1} + \vec{p}_{p1} = \vec{p}_{e2} + \vec{p}_{p2}$$

$$\therefore \vec{p}_{p2} = (\vec{p}_{e1} + \vec{p}_{p1} - \vec{p}_{e2})$$

$$\therefore -m_p^2 = -m_e^2 - m_p^2 - m_e^2 + 2\vec{p}_{e1} \cdot \vec{p}_{p1}$$

$$0 = 2\vec{p}_{e1} \cdot \vec{p}_{e2} - 2\vec{p}_{p1} \cdot \vec{p}_{e2}$$

$$0 = -2m_e^2 + 2 \begin{pmatrix} E_{e1} \\ \vec{p}_{e1} \end{pmatrix} \cdot \begin{pmatrix} E_{e2} \\ \vec{p}_{e2} \end{pmatrix} + 2 \begin{pmatrix} E_{e1} \\ \vec{p}_{e1} \end{pmatrix} \cdot \begin{pmatrix} m_p \\ 0 \end{pmatrix} - 2 \begin{pmatrix} m_p \\ 0 \end{pmatrix} \cdot \begin{pmatrix} E_{e2} \\ \vec{p}_{e2} \end{pmatrix}$$

$$= -2m_e^2 + 2E_{e1}E_{e2} - 2p_{e1}p_{e2}\cos\theta$$

$$- 2E_{e1}m_p + 2E_{e2}m_p$$

$$\therefore E_{e1} = 1 \text{ GeV} = 1000 \text{ MeV}$$

$$E_{e2} = 800 \text{ MeV} \quad , m_e = 0.511 \text{ MeV}/c^2$$

$$\therefore p_{e1} \approx 1000 \text{ MeV}/c \approx E_{e1}$$

$$p_{e2} \approx 800 \text{ MeV}/c \approx E_{e2}$$

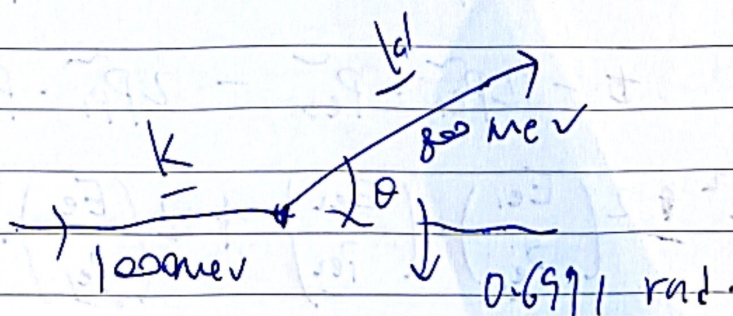
$$\therefore (E_{e1} + E_{e2})m_p + m_e^2 = (E_{e1}E_{e2})(1 - \cos\theta)$$

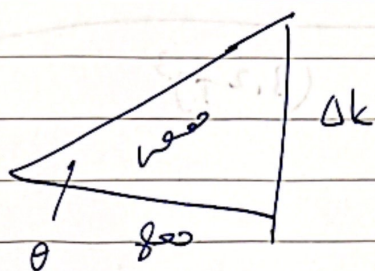
$$\therefore 1 - \cos\theta \approx \frac{(E_{e1} + E_{e2})m_p}{E_{e1}E_{e2}}$$

$$\cos\theta \approx 1 - \frac{(E_{e1} + E_{e2})m_p}{E_{e1}E_{e2}} = 1 - 0.2346$$

$$= 0.7654$$

$$\theta \approx \arccos(0.7654) = 0.6991 \text{ rad}$$





$$\Delta k = \sqrt{1000^2 + 800^2 - 2 \cdot 1000 \cdot 800 \cdot \cos(0.6991)}$$

$$= 644 \frac{\text{MeV}}{c \cdot \hbar}$$

$$c \cdot \hbar = (3 \times 10^8) (1.05 \times 10^{-34})$$

$$= 3.15 \times 10^{-26} \text{ J} \cdot \text{s} \cdot \text{m/s}$$

$$= 1.997 \times 10^{-7} \text{ eV} \cdot \text{m}$$

$$= 1.997 \times 10^{-13} \text{ MeV} \cdot \text{m}$$

$$= \cancel{1997 \text{ MeV} \cdot \text{fm}} \quad | \quad 197 \text{ MeV} \cdot \text{fm}$$

$$(\alpha_{\text{fm}} = 1.44 \text{ MeV} \cdot \text{fm})$$

$$\therefore \Delta k = 3.27 \text{ fm}^{-1}$$

$$\sin(\Delta k a) = -0.128$$

$$\Delta k a = 3.27 \text{ fm}^{-1} \times \text{fm}$$

$$\cos(\Delta k a) = -0.992$$

$$\therefore |F(\Delta k)|^2 = \left| \frac{3(-0.128 - (3.27)(-0.992))}{(3.27)^3} \right|^2$$

$$= 0.0715$$

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{Rutherford}} = \frac{Z^2 \alpha_{\text{EM}}^2}{4E^2 \sin^4(\theta/2)}$$

$$= \frac{(1)^2 (1.44)^2}{4(1000)^2 \sin^4\left(\frac{0.6991}{2}\right)}$$

$$= 3.77 \times 10^{-5} \text{ fm}^2$$

$$\therefore \frac{d\sigma}{d\Omega} = 3.77 \times 10^{-5} \times 0.0715$$

$$= \underline{\underline{2.694 \times 10^{-6} \text{ fm}^2}}$$

As energy increases, the frequency of the light increases. The wavelength of the light decreases. The order of the spectrum is: violet, blue, green, yellow, orange, red. The visible spectrum is a small part of the electromagnetic spectrum. The wavelength of the light is measured in meters. The frequency of the light is measured in Hertz. The speed of light is constant in a vacuum.

Strong interaction affects the charge

Electrons scatter off from protons

$$F = k \frac{q_1 q_2}{r^2}$$

$$F = k \frac{q_1 q_2}{r^2}$$

$$F = k \frac{q_1 q_2}{r^2}$$

(d) - As energy increases, de Broglie wave length of electron decreases to the order of nuclear size. We can have deep inelastic scattering where electrons scatter off partons inside the nucleus (proton). This means that the charge density of proton is clearly not uniform.

- strong interaction affects the charge distribution.

- electrons scatter off from quarks

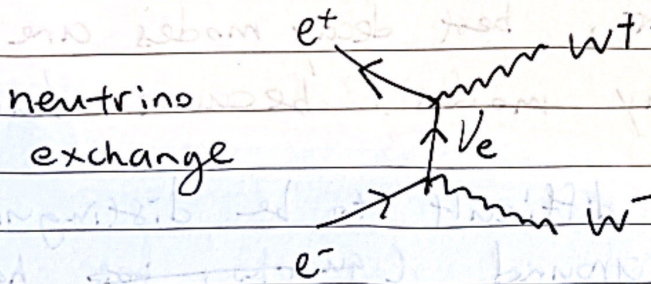
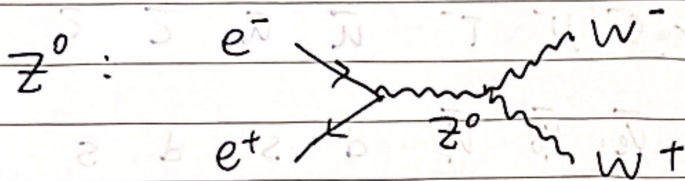
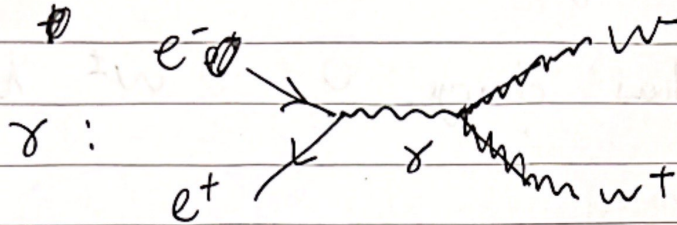
$$q(u) = \frac{2}{3}$$

$$q(d) = -\frac{1}{3}$$

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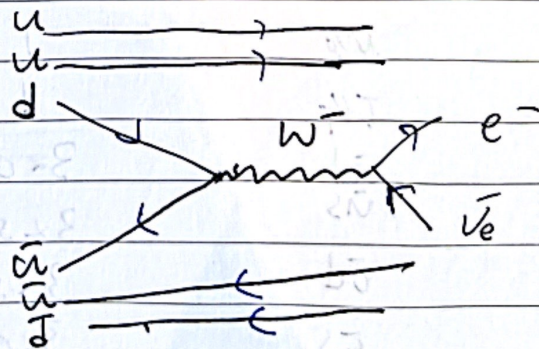
4(a)

$$e^+ + e^- \rightarrow W^+ + W^-$$



$$p + \bar{p} \rightarrow W + X$$

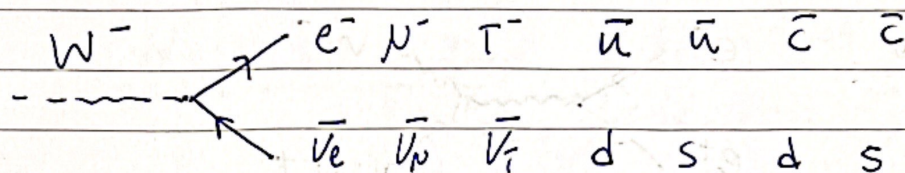
$\downarrow$ $\downarrow$
uud $\bar{u}\bar{u}\bar{d}$



in electron-positron collider, single W boson cannot be produced because this violates charge conservation

e^+e^- has charge 0, $W^\pm$ has charge ± 1

(b) W -boson decay



In $p\bar{p}$ collider, best decay modes are the leptonic decay modes, because hadronic modes are difficult to be distinguished from background (~~quarks, jets~~ hadron jets)

lepton universality: $W \rightarrow l\bar{l}$

mode fraction (θ_c = Cabibbo angle)

$e\bar{e}$

1

$\mu\bar{\mu}$

1

$\tau\bar{\tau}$

1

$\bar{u}d$

$3 \times \cos^2 \theta_c$

$\bar{u}s$

$3 \times \sin^2 \theta_c$

$\bar{c}d$

$3 \times \sin^2 \theta_c$

$\bar{c}s$

$3 \times \cos^2 \theta_c$

3 = color degeneracy

$\therefore$ Branching fraction

$Br(W \rightarrow l\bar{l}) = \frac{1}{1+1+1+3+3} = \frac{1}{9}$

c c)

~~σ_W~~

events of inelastic collision

At 1.96 TeV

$$N_{\text{inelastic}} = 3 \times 10^{14}$$

$$\sigma_W = 25 \text{ nb}$$

$$\sigma_{\text{inelastic}} = 30 \text{ mb}$$

$\therefore$ number of W boson events

$$N_W = N_{\text{inelastic}} \frac{\sigma_W}{\sigma_{\text{inelastic}}}$$

$$= \frac{25 \text{ nb}}{30 \text{ mb}} \times 3 \times 10^{14}$$

$$= \underline{\underline{2.5 \times 10^8}}$$

of W boson in best decay modes

$$N_{\text{leptonic}} = \left(\frac{1}{9} + \frac{1}{9} + \frac{1}{9} \right) \times 2.5 \times 10^8$$

$\downarrow \quad \downarrow \quad \downarrow$
 $e \quad \mu \quad \tau$

$$= \underline{\underline{8.33 \times 10^7}}$$

(d) CM energy $\sqrt{s} = 2 \text{ TeV}$

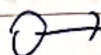
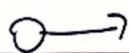
$c = \hbar = 1$

Assume ultrarelativistic

$$E \approx p$$

quark 1

ant. quark 2



$$\tilde{P}_1 = \begin{pmatrix} x_p P_p \\ x_{\bar{p}} P_{\bar{p}} \\ 0 \\ 0 \end{pmatrix}$$

$$\tilde{P}_2 = \begin{pmatrix} x_{\bar{p}} P_{\bar{p}} \\ -x_p P_p \\ 0 \\ 0 \end{pmatrix}$$

$$\tilde{P}_W = \begin{pmatrix} E_W \\ P_W \\ 0 \\ 0 \end{pmatrix}$$

4 vector momentum conservation.

$$(P_W)^2 = (P_1 + P_2)^2 \quad \eta = \text{diag}(-1, 1, 1, 1)$$

$$\therefore -m_W^2 = -m_q^2 - m_{\bar{q}}^2 + 2 \tilde{P}_1 \cdot \tilde{P}_2$$

$$-m_W^2 = -m_q^2 - m_{\bar{q}}^2 + 2(-x_p x_{\bar{p}} P_p P_{\bar{p}} - x_{\bar{p}} x_p P_{\bar{p}} P_p)$$

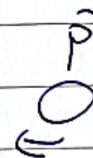
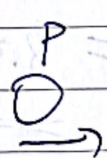
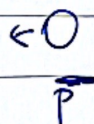
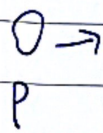
$$m_q, m_{\bar{q}} \ll m_W$$

$$\therefore m_W^2 = 4 x_p x_{\bar{p}} P_p P_{\bar{p}}$$

lab frame

~~lab frame~~

CM frame



$$\tilde{P}_p = \begin{pmatrix} P_p \\ P_p \\ 0 \\ 0 \end{pmatrix}$$

$$\tilde{P}_{\bar{p}} = \begin{pmatrix} P_{\bar{p}} \\ -P_{\bar{p}} \\ 0 \\ 0 \end{pmatrix}$$

$$\tilde{P}_{\text{tot}} = \tilde{P}_p + \tilde{P}_{\bar{p}} =$$

= total 4 momentum

$$\therefore \vec{P}_{tot} \cdot \vec{P}_{tot} = -(P_p + P_{\bar{p}})^2 + (P_{\bar{p}} - P_p)^2$$

$$= -4P_p P_{\bar{p}}$$

in cm frame, invariant of dot product

$\vec{P}_{tot} \cdot \vec{P}_{tot}$ gives

$$\vec{P}_{tot}' \cdot \vec{P}_{tot}' = \begin{pmatrix} E_{cm} \\ 0 \end{pmatrix} \cdot \begin{pmatrix} E_{cm} \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} S \\ 0 \end{pmatrix} \cdot \begin{pmatrix} S \\ 0 \end{pmatrix} = -S^2$$

$$\therefore -S^2 = -P_p P_{\bar{p}} \times 4 \quad \underline{S^2 = 4P_p P_{\bar{p}}}$$

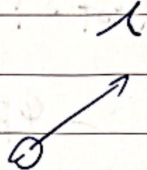
$$\therefore M_W^2 = X_p X_{\bar{p}} S^2$$

$$\therefore X_p X_{\bar{p}} = \frac{m_W^2}{S^2} = \left(\frac{80 \text{ GeV}}{2 \text{ TeV}} \right)^2 = \frac{1}{625}$$

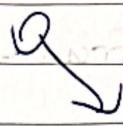
If $X = X_p = X_{\bar{p}} \quad X^2 = \frac{1}{625} \quad \therefore X = \frac{1}{25}$

(e)

$W \rightarrow$



$$\tilde{p}^l = \begin{pmatrix} E^l \\ p_x^l \\ p_y^l \\ p_z^l \end{pmatrix}$$



$$\tilde{p}^{\nu} = \begin{pmatrix} E^{\nu} \\ p_x^{\nu} \\ p_y^{\nu} \\ p_z^{\nu} \end{pmatrix}$$

$$\tilde{p}_{tot} = \tilde{p}^l + \tilde{p}^{\nu}$$

$$\tilde{p}_{tot} \cdot \tilde{p}_{tot} = \tilde{p}_{tot}^{cm} \cdot \tilde{p}_{tot}^{cm} =$$

total 4 momentum in centre of mass frame

$$= \begin{pmatrix} E^{cm} \\ \underline{0} \end{pmatrix} \cdot \begin{pmatrix} E^{cm} \\ \underline{0} \end{pmatrix} = \begin{pmatrix} S \\ \underline{0} \end{pmatrix} \cdot \begin{pmatrix} S \\ \underline{0} \end{pmatrix} = -S^2$$

$$\therefore -S^2 = (\tilde{p}^l + \tilde{p}^{\nu})^2$$

$$= -m_l^2 - m_{\nu}^2 + 2\tilde{p}^l \cdot \tilde{p}^{\nu}$$

↓
lepton mass
(ignore)

↓
neutrino
mass
(≈ 0)

$$\therefore -S^2 \approx 2\tilde{p}^l \cdot \tilde{p}^{\nu} = 2 \begin{pmatrix} E^l \\ p_x^l \\ p_y^l \\ p_z^l \end{pmatrix} \cdot \begin{pmatrix} E^{\nu} \\ p_x^{\nu} \\ p_y^{\nu} \\ p_z^{\nu} \end{pmatrix}$$

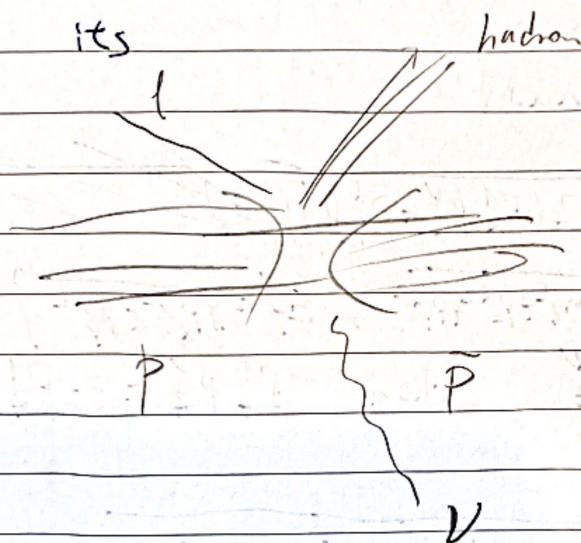
$$m_T^2 = (P_{Tl} + P_{T\nu})^2$$

$$\therefore S^2 = 2E^l E^\nu - 2(P_x^l P_x^\nu + P_y^l P_y^\nu + P_z^l P_z^\nu)$$

invariant mass

$$m = \sqrt{S^2} = \sqrt{2(E^l E^\nu - (P_x^l P_x^\nu + P_y^l P_y^\nu + P_z^l P_z^\nu))}$$

(f) neutrino cannot be detected, so
its



~~trans~~ momentum
is measured by
measuring the ~~momenta~~
momenta of ~~the~~ other
products

- only ~~longitudinal~~ transverse momentum of neutrino can be measured because we know for sure the initial total transverse momentum is 0, once ~~we have~~ we've measured ~~the~~ at the momenta of ~~the~~ leptons ~~other final products~~, we can use conservation of momentum along the transverse direction to determine the transverse momentum of neutrino.
- The longitudinal momentum of neutrino cannot be measured because we don't know the fraction of $p, \bar{p}$ momentum are carried by the

quark - antiquark that collide to produce
a lepton - neutrino. (the $x_p, x_{\bar{p}}$ in (d))

(not possible to put a detector in the longitudinal
direction ??)

(interaction with vacuum ??)

(g)

transverse mass

$$m_T = \sqrt{2E_T^l E_T^\nu - 2(p_x^l p_x^\nu + p_y^l p_y^\nu)} = \text{~~scribbled out~~}$$

$$(E_T^l = \sqrt{m_l^2 + p_x^{l2} + p_y^{l2}}, E_T^\nu = \sqrt{p_x^{\nu 2} + p_y^{\nu 2}}) \quad (m_l = 0)$$

(h) if $p_z^l = -p_z^\nu$, then $p_z^l p_z^\nu = -p_z^{l2} < 0$

$$\therefore m = \sqrt{2E^l E^\nu - 2(p_x^l p_x^\nu + p_y^l p_y^\nu + p_z^l p_z^\nu)} \quad \left(\begin{array}{l} \because E^l > E_T^l \\ E^\nu > E_T^\nu \end{array} \right)$$

$$\Rightarrow \sqrt{m_T^2 - \underbrace{2p_z^l p_z^\nu}_{< 0}} > m_T$$

$\therefore$ transverse mass less than invariant mass.