BPhO 2019 Round 1
Solutions

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1 Section 1

Problem 1.

Solutions. (a) In SI units, $u = 60$, $\theta = 40°$, so at $t = 3$

$$v_x = u \cos \theta = 46.0 \, \text{m/s} \tag{1}$$

and

$$v_y = u \sin \theta - gt = 9.14 \, \text{m/s} \tag{2}$$

The position

$$x = v_x t = 138 \, \text{m} \tag{3}$$

and

$$y = u \sin \theta \, t - \frac{1}{2} gt^2 = 71.6 \, \text{m} \tag{4}$$

(b) since $\boldsymbol{s} = (2t, 6t)$, the velocity

$$\boldsymbol{v} = \dot{\boldsymbol{s}} = (2, 6) \tag{5}$$

So speed

$$v = |\boldsymbol{v}| = \sqrt{2^2 + 6^2} = 6.3 \, \text{m} \tag{6}$$

The bearing is

$$\theta = \tan(\frac{2}{6}) = 18.4° \tag{7}$$

Since $\boldsymbol{v}$ is independent of t, we have constant velocity motion and the acceleration is 0.

(c) The area density of a piece of paper is roughly $\sigma = 100 \, \text{g/m}^2$, the size of a pinhead is about $A = 1 \, \text{mm} \times 1 \, \text{mm}$. So the mass is

$$m = \sigma A = 10^{-7} \, \text{kg} \tag{8}$$

(d) From the equation we have

$$[v]^2 = [a][\lambda] = [b][\rho]^{-1}[\lambda]^{-1} \tag{9}$$

So

$$L^2 T^{-2} = [a]L = [b]M^{-1}L^3 L^{-1} \tag{10}$$

Therefore

$$[a] = LT^{-2} = \text{m/s}^2 \tag{11}$$

and

$$[b] = MT^{-2} = \text{kg/s}^2 \tag{12}$$

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(e) The resistance of aluminium is

$$R_a = \rho_a \frac{l}{6A} \tag{13}$$

and that of steel is

$$R_s = \rho_s \frac{l}{A} \tag{14}$$

The total resistance is

$$R = \frac{R_a R_s}{R_a + R_s} = \frac{\rho_a \rho_s l}{(\rho_a + 6\rho_s)A} \tag{15}$$

Change in resistance

$$\Delta R = R_a - R = \frac{\rho_a^2 l}{6(\rho_a + 6\rho_s)A} \tag{16}$$

Solve for l gives $l = 505\,\mathrm{m}$

(f) We want the combined density of Platinum and Potassium equal to the density of Mercury when it just about to sink.

$$\frac{\rho_{\mathrm{Pt}} V_{\mathrm{Pt}} + \rho_{\mathrm{K}} V_{\mathrm{K}}}{V_{\mathrm{Pt}} + V_{\mathrm{K}}} = \rho_{\mathrm{Hg}} \tag{17}$$

Solve this equation yields $V_{\mathrm{Pt}} = 16.1\,\mathrm{cm}^3$

(g) The heat absorbed by ice plus the heat absorbed by the water melted from ice equals to the heat released by the original water. In the final state all water has the same final temperature T_f, therefore

$$L_{\mathrm{ice}} \cdot \frac{1}{2} m_{\mathrm{ice}} + \frac{1}{2} \cdot m_{\mathrm{ice}} c_{\mathrm{ice}} (T_f - 0) = (V_{\mathrm{total}} - \frac{m_{\mathrm{ice}}}{\rho_{\mathrm{ice}}}) \rho_{\mathrm{water}} c_{\mathrm{water}} (T_i - T_f) \tag{18}$$

T_f is only unknown in the above equation so we can solve for it to get $T_f = 4.25\,^\circ\mathrm{C}$

(h) The velocity vectors are shown in the figure , where F, C, P, and W represent ferry, container
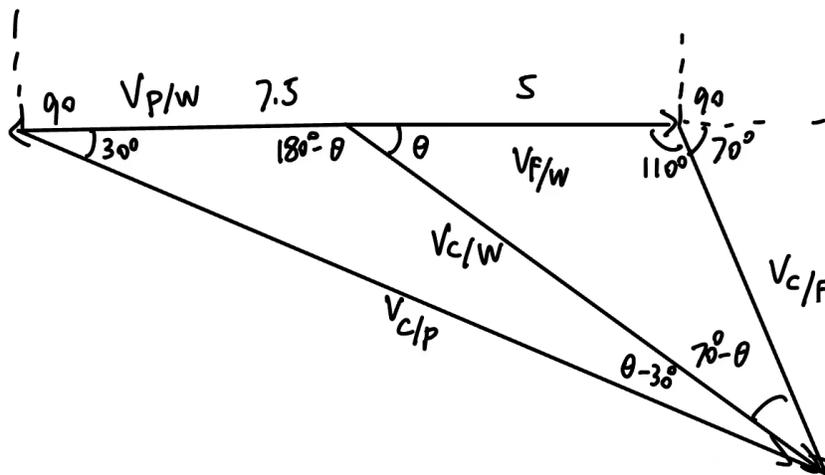

Figure 1: (h)

ship, pilot boat, and water respectively. The velocities $\boldsymbol{v}_{A/B}$ represent the velocity of A relative to B.

let $v = v_{C/W}$, which represents the speed of the container ship, and denote the angle θ as shown in the figure.

Using sine law we have

$$\frac{v}{\sin 30°} = \frac{7.5}{\sin(\theta - 30°)} \tag{19}$$

and

$$\frac{v}{\sin 110°} = \frac{5}{\sin(70° - \theta)} \tag{20}$$

Dividing the two equations and using angle sum identities for trigonometric functions we arrive at

$$(7.5)(\sin 30°)(\sin 70° \cos \theta - \cos 70° \sin \theta) = (5)(\sin 110°)(\sin \theta \cos 30° - \cos \theta \sin 30°) \tag{21}$$

which gives $\tan \theta = 1.0976$ so $\theta = 47.7°$. Hence the bearing equals to $\theta + 90° = 137.7°$ Substitute θ back into any of the previous equations gives $v = 12.3$ m/s.

(i) Since $P = Fv$ is constant, using Newton's Second Law and chain rule we have

$$P = Fv = mav = m(\frac{\mathrm{d}v}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t})v = m(v\frac{\mathrm{d}v}{\mathrm{d}x})v \tag{22}$$

is a constant. Because the car accelerates from rest at position $x = 0$,

$$\int_0^v v'^2 \mathrm{d}v' = \frac{P}{m} \int_0^x \mathrm{d}x' \tag{23}$$

So,

$$v(x) = \left(\frac{3Px}{m}\right)^{\frac{1}{3}} \tag{24}$$

(j) Using the work energy principle, the work done by the buoyancy force (upthrust) F_b equals to the change in total kinetic and potential energy. At maximum height the velocity is 0 so there is no change in kinetic energy. We have

$$F_b = \rho g \cdot \frac{4}{3}\pi r^3 \tag{25}$$

and that

$$F_b d = mg(d + h) \tag{26}$$

Hence rearranging gives

$$\frac{h}{d} = \frac{4\pi r^3 \rho}{3m} - 1 \tag{27}$$

(k) The units of the equation in the question gives

$$MT^{-1} = (ML^{-3})^\alpha (L^2)^\beta (LT^{-2})^\gamma \tag{28}$$

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Equating the left and right gives

$$\alpha = 1$$
$$-2\gamma = -1 \tag{29}$$
$$-3\alpha + 2\beta + \gamma = 0$$

So solve this lead to

$$\alpha = 1, \quad \beta = \frac{5}{4}, \quad \gamma = \frac{1}{2} \tag{30}$$

So the original equation becomes

$$\frac{\mathrm{d}m}{\mathrm{d}t} = k\rho A^{\frac{5}{4}} g^{\frac{1}{2}} \tag{31}$$

The same sand timer means ρ and A are the same. Since the rate scales as the square root of g, and total time taken scales as the inverse of the rate

$$T_{\text{Moon}} = \sqrt{\frac{9.8}{1.6}} T_{\text{Earth}} = 37.1 \, \text{mins} \tag{32}$$

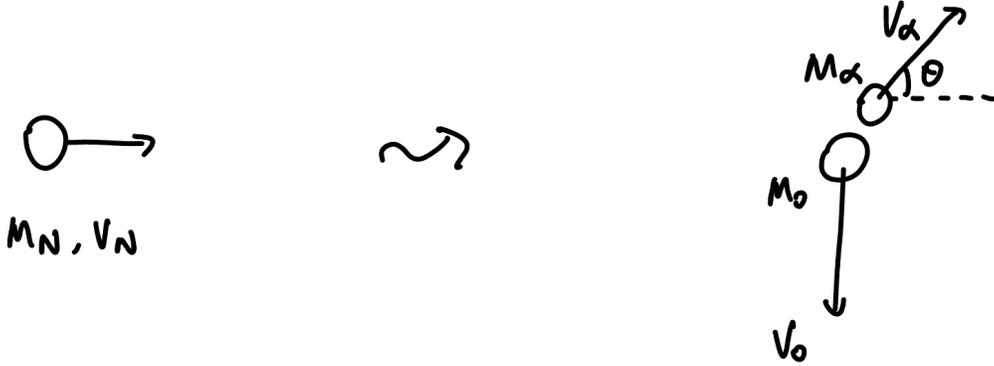(1) The collision process is shown in the figure.

Figure 2: (1)

Conservation of momentum in x-direction

$$M_n v_n = M_\alpha v_\alpha \cos\theta \tag{33}$$

Conservation of momentum in y-direction

$$M_o v_o = M_\alpha v_\alpha \sin\theta \tag{34}$$

The event produces an extra kinetic energy $E_0 = 6.25$ MeV. So conservation of energy

$$\frac{1}{2} M_n v_n^2 + E_0 = \frac{1}{2} M_\alpha v_\alpha^2 + \frac{1}{2} M_o v_o^2 \tag{35}$$

, where the masses are $M_n = 20m_u$, $M_\alpha = 4m_u$, $M_o = 16m_u$ for the atomic mass unit $m_u = 1.66 \times 10^{-27}$ kg. And $v_n = 3.0 \times 10^6$ m/s.

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We have 3 equations for 3 unknowns v_α, v_o, and θ. The system of equations can be solved. We just want v_α for this question.

To eliminate θ, we square the two momentum equations and add them up.

$$M_\alpha^2 v_\alpha^2(\cos^2\theta + \sin^2\theta) = M_\alpha^2 v_\alpha^2 = M_n^2 v_n^2 + M_o^2 v_o^2 \tag{36}$$

where we have used $\cos^2\theta + \sin^2\theta = 1$ So

$$\frac{1}{2}M_o v_o^2 = \frac{M_\alpha^2 v_\alpha^2 - M_n^2 v_n^2}{2M_o} \tag{37}$$

Substituting this into the energy equation elimates v_o and gives

$$\frac{1}{2}(1 + \frac{M_\alpha}{M_o})M_\alpha v_\alpha^2 = \frac{1}{2}(1 + \frac{M_n}{M_o})M_n v_n^2 + E_0 \tag{38}$$

This is an equation that involves only v_α. Solve it gives $v_\alpha = 1.8 \times 10^7 \mathrm{m/s}$

(m) We use the fact that the Earth rotates from West to East. Let this angular velocity be Ω. It can be calculated that

$$\Omega = \frac{2\pi}{24 \times 60 \times 60} = 7.27 \times 10^{-5}\,\mathrm{rad/s} \tag{39}$$

The airplane flies at a velocity $v = 250$ m/s relative to the Earth. The plane flies at a low altitude, which means that we can approximate the distance from the centre of the Earth to the plane as the radius of Earth, denoted by $R = 6.37 \times 10^6$ m. Since the plane flies along the equator, the angular speed of the plane relative to Earth is $\omega' = \frac{v}{R} = 3.92 \times 10^{-5}$ rad/s. Let the masses of the Earth and the plane be M and m respectively. From the question $m = 1$ kg

In the inertial reference frame of deep space, if the airplane flies from West to East, its angular velocity is $\omega_1 = \Omega + \omega'$. Note that the apparent weight is the normal force provided by the weight scale, hence by Newton's Second Law,

$$\frac{GMm}{R^2} - W_1 = mR(\Omega + \omega')^2 \tag{40}$$

where we took inwards towards Earth to be the positive direction. If the airplane flies from East to West, its angular velocity is $\omega_2 = \Omega - \omega'$, so we have

$$\frac{GMm}{R^2} - W_2 = mR(\Omega - \omega')^2 \tag{41}$$

Hence the difference in apparent weight,

$$\Delta W = W_2 - W_1 = mR(\Omega + \omega')^2 - mR(\Omega - \omega')^2 \tag{42}$$

so using $a^2 - b^2 = (a + b)(a - b)$

$$\Delta W = 4mR\omega'\Omega = 0.073\,\mathrm{N} \tag{43}$$

Note that this question is much more easily done in this inertial frame of reference. If we go to the Earth's frame of reference, the plane experiences both the centrifugal force $\mathbf{F}_{cf} = -2m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{R}) = mR\Omega^2$ outwards from Earth, and the Coriolis force $\mathbf{F}_{cor} = -2m\boldsymbol{\Omega} \times \boldsymbol{v} = \pm 2m\Omega v = \pm 2mR\Omega\omega'$ outwards from Earth, where plus if flying due East and minus if flying

due West. So the Newton's Second Law equation becomes $\frac{GMm}{R^2} - W - F_{cf} - F_{cor} = mR\omega'^2$, where we take inwards towards Earth to be the positive direction and $R\omega'^2$ is the centripetal acceleration of the plane in Earth's frame. This can recover the previous results but is obviously much more tedious. If you transfer to the rest frame of the plane then the issue is not that complicated as there is only the centrifugal force.

(n) Let $V = 75\,\mathrm{cm}^3$, then $2V = 150\,\mathrm{cm}^3$. At the final state in equilibrium the pressure is the same across the two bulbs. Using the ideal gas equation $PV = Nk_BT$ and the fact that the total number of gas particles in the two glass bulbs remains the same,

$$\frac{P_i(3V)}{k_bT_i} = \frac{P_f(2V)}{k_BT_i} + \frac{P_fV}{k_BT_f} \tag{44}$$

, keeping in mind that the temperatures need to be converted to Kelvins. We arrive at $P_f = 9.48 \times 10^4$ Pa.
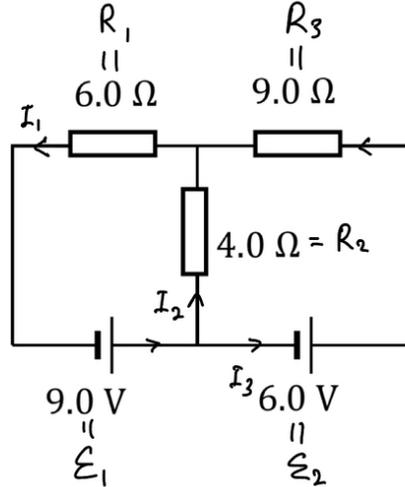
(o) According to the figure, using Kirchhoff's Laws,

Figure 3: (o)

$$\mathcal{E}_1 - I_2R_2 - I_1R_1 = 0$$
$$\mathcal{E}_2 - I_3R_3 + I_2R_2 = 0 \tag{45}$$
$$I_1 - I_2 - I_3 = 0$$

We have 3 equations for 3 unknowns I_1, I_2, and I_3. This can be solved. Solving the system of equations gives $I_1 = \frac{47}{38} = 1.24$ A.

(p) This question tests the knowledge of both the photoelectric effect and electrostatics. As more and more electrons escape from the metal surface due to the illumination of light, excess positive charges will develop at the surface of the copper sphere, giving the sphere a potential $V = \frac{Q}{4\pi\varepsilon_0r}$. Then each new electron escaping from the metal surface will possess an electrostatic potential energy $U = -eV$, where e is the usual unit charge. The maximum

value of V occurs when the magnitude of this potential energy U equals to the maximum kinetic energy possessed by the escaped photoelectron. That is

$$\frac{1}{2}mv^2_{\text{max}} = \frac{hc}{\lambda} - \phi = eV \tag{46}$$

Solving this gives $V = 3.79$ V. The number of electron lost equals to the total charge Q that produces the maximum V divided by the magnitude of charge of one electron.

$$N = \frac{4\pi\varepsilon_0 rV}{e} = 5.3 \times 10^6 \tag{47}$$

(q) When the three spheres are connected by conducting wires and then reaches equilibrium, the potential of these 3 spheres must be equal. They are well separated means that they don't induce charges on each other. Also, due to charge conservation the total charge of the 3 spheres must be same before and after the wire connection. So we have the following equation

$$4\pi\varepsilon_0(\frac{1}{3}R)V + 0 + 4\pi\varepsilon_0(R)V = 4\pi\varepsilon_0(\frac{1}{3}R)V' + 4\pi\varepsilon_0(\frac{1}{2}R)V' + 4\pi\varepsilon_0(R)V' \tag{48}$$

Solve this gives $V' = \frac{8}{11}V$. So charge on the second sphere

$$Q_2 = 4\pi\varepsilon_0(\frac{1}{2}R)\frac{8}{11}V \tag{49}$$

Total charge is

$$Q = 4\pi\varepsilon_0(\frac{1}{3}R)V + 4\pi\varepsilon_0(R)V \tag{50}$$

The required fraction is

$$\frac{Q_2}{Q} = \frac{3}{11} \tag{51}$$

(r) The heat flux is defined as

$$J = \frac{1}{A}\frac{dQ}{dt} = \kappa\frac{d\theta}{dx} \tag{52}$$

in this very special 1-D situation, where κ is the thermal conductivity.

For a specific volume along the rod, $\frac{dQ_{\text{vol}}}{dt} = J_1 A - J_2 A$ when the area is a constant. So if this 1-D constant area system is in steady state, there should be no change in heat flux into versus out of the volume, so $J_1 = J_2$. This also applies at the junctions. Hence,

$$\kappa_1\frac{100 - \theta_1}{l} = \kappa_2\frac{\theta_1 - \theta_2}{l} = \kappa_3\frac{\theta_2 - 0}{l} \tag{53}$$

Solve this we get $\theta_1 = 28.6\,°\text{C}$ to be the temperature of the steel/copper junction, and $\theta_2 = 17.9\,°\text{C}$ to be the temperature of the copper/aluminium junction.

(s) The popping sound indicates the formation of a stationary wave in the bump. At the moment when the cork is fired, this is a open pipe. The relationship between the wavelength and the length of the tube at the moment of firing is given by,

$$l = \frac{2n - 1}{4}\lambda \tag{54}$$

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for n a positive integer. The wavelength is

$$\lambda = \frac{v}{f} = 0.64\,\text{m} \tag{55}$$

We know that l cannot be greater than $l_0 = 0.25$ m, which is the original length of the tube, so the only allowed n is $n = 1$.

The frequency stationary wave this should equal to the length of the bump after compression. So

$$l = \frac{\lambda}{4} = 0.161\,\text{m} \tag{56}$$

From ideal gas equation at constant temperature and particle number we have

$$p_0 A l_0 = P A l \tag{57}$$

So $P = 1.55 \times 10^5$ Pa. The required force exerted by the person is

$$F = (P - p_0)A = 22\,\text{N} \tag{58}$$

(t) We have

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -k(T - T_0) \tag{59}$$

so

$$\int_{T_i}^{T_f} \frac{\mathrm{d}T}{T - T_0} = -k \int_0^t \mathrm{d}t' \tag{60}$$

which gives

$$\ln \frac{T_f - T_0}{T_i - T_0} = -kt \tag{61}$$

hence

$$T_f = T_0 + (T_i - T_0)\mathrm{e}^{-kt} \tag{62}$$

Using the initial conditions given in the questions we have

$$\begin{aligned}
29.7 &= 20.0 + (30.2 - 20.0)\mathrm{e}^{-k \cdot 1\text{min}} \\
23.0 &= 20.0 + (24.0 - 20.0)\mathrm{e}^{-k\Delta t}
\end{aligned} \tag{63}$$

We have two equations and two unknowns, which can be solved. The result is $\Delta t = 5.7$ min. $\square$

2 Section 2

Problem 2.

Problem 3.

solutions.   (a) The direction of the particle's velocity is constantly changing, so the particle has non-zero acceleration. By Newton's Second Law, the resultant force acting on the particle must be non-zero.
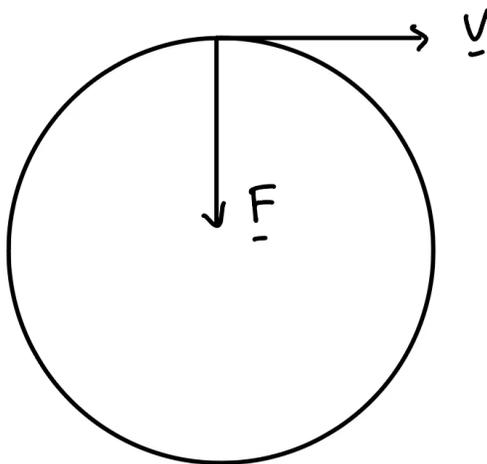
Figure 4: 3.(a)

Its magnitude

$$F = m\frac{v^2}{r} = m\omega^2 r \tag{64}$$

(b) In the z-direction there is no acceleration,

$$T\cos\theta = mg \tag{65}$$

In the radial direction the component of tension provides the centripetal force

$$T\sin\theta = m\omega^2(a + l\sin\theta) \tag{66}$$

Eliminate T gives

$$g\tan\theta = \omega^2(a + l\sin\theta) \tag{67}$$

Hence

$$\omega = \sqrt{\frac{g\tan\theta}{a + l\sin\theta}} \tag{68}$$

Substitute in numerical values gives $\omega = 0.84$ rad/s.
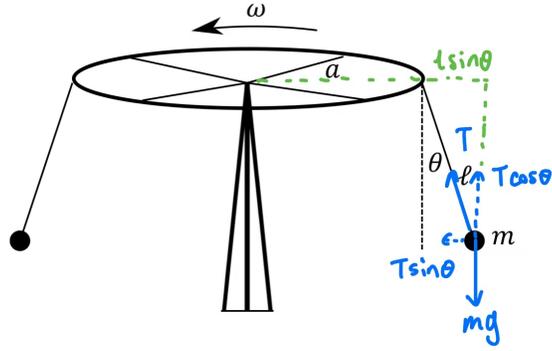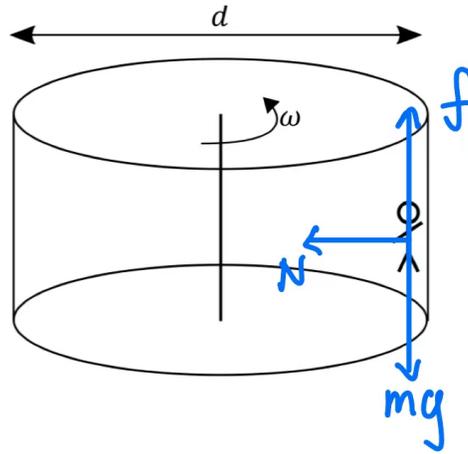
(c)

Figure 5: 3.(b)

Figure 6: 3.(c)i

i As shown in figure

ii In the z-direction, static friction $f = mg$.

The normal force N provides the centripetal acceleration

$$N = m\omega^2 \frac{d}{2} \tag{69}$$

Static friction staisfies $f \leq \mu N$ so

$$N = m\omega^2 \frac{d}{2} \geq \frac{f}{\mu} = \frac{mg}{\mu} \tag{70}$$

which gives

$$\omega^2 \geq \frac{2g}{\mu d} \tag{71}$$

So minimum ω is given by

$$\omega = \frac{2g}{\mu d} \tag{72}$$

Substitute in numerical values gives $\omega = 2.2$ rad/s

iii As shown in the figure
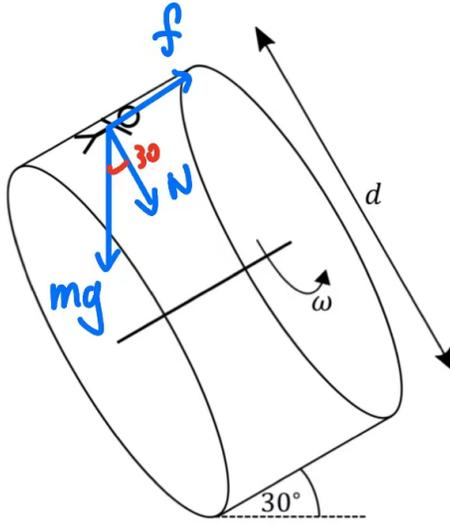

Figure 7: 3.(c)iii

iv Along the wall

$$f = mg \sin 30° = \frac{1}{2} mg \tag{73}$$

Perpendicular to the wall

$$N + mg \cos 30° = m\omega_t^2 \frac{d}{2} \tag{74}$$

As minimum ω_t, $f = \mu N$. Also $\cos 30° = \frac{\sqrt{3}}{2}$. We have

$$\frac{1}{2\mu} mg + \frac{\sqrt{3}}{2} mg = m\omega_t^2 \frac{d}{2} \tag{75}$$

So

$$\omega = \sqrt{\frac{(1 + \sqrt{3}\mu)g}{\mu d}} \tag{76}$$

Substitute in numerical values yields $\omega = 2.0$ rad/s

(d)

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i By energy conservation, the speed at the top of the circle v is given by

$$\frac{1}{2}mv^2 = mg(h - 2r) \tag{77}$$

so

$$v^2 = 2g(h - 2r) \tag{78}$$

At the top of the circle the normal force and the gravity together provide the centripetal force if the car was to stay on the track. In the limiting case when the car is just able to stay on track, the normal force is zero and the gravity alone must provide the centripetal force needed.

$$mg = m\frac{v^2}{r} \tag{79}$$

so

$$gr = v^2 = 2g(h - 2r) \tag{80}$$

Hence the minimum height is

$$h = \frac{5}{2}r = 50\,\text{m} \tag{81}$$
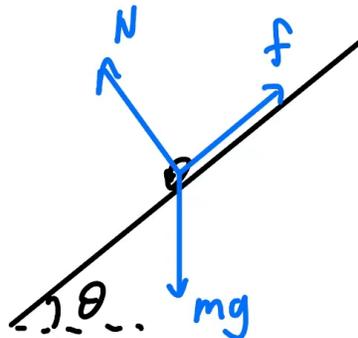
ii As shown in the figure

Figure 8: 3.(d)ii(i)

Using the work-energy principle, which states that the total work done by non-conservative forces equals to the change in potential energy plus change in kinetic energy.

$$-fl = \frac{1}{2}mv^2 - mgl\sin\theta \tag{82}$$

where $\theta = 40°$. The friction is kinetic so

$$f = \mu N \tag{83}$$

And force balance in the direction perpendicular to the tract gives

$$N = mg\cos\theta \tag{84}$$

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Together gives
$$v^2 = 2gl(\sin\theta - \mu\cos\theta) \tag{85}$$

Taking $\mu = 0$ gives the speed would be if there is no friction.

$$v'^2 = 2gl\sin\theta \tag{86}$$

fraction loss in speed is

$$\frac{\Delta v}{v'} = \frac{v' - v}{v'} = 1 - \frac{v}{v'} = 1 - \sqrt{1 - \mu\cot\theta} = 3.0\% \tag{87}$$

iii As the car enters the track, the change in normal force is equal to the centripetal force

$$\Delta N = m\frac{v^2}{l/2} = \frac{4mgl\sin\theta}{l} = 4mg\sin\theta = 1500\,\text{N} \tag{88}$$

(e)

  i Conservation of energy

$$\frac{1}{2}(2M)v^2 = (2M)gh \tag{89}$$

    gives

$$v = \sqrt{2gh} \tag{90}$$

  ii Taking the ground to be 0 potential energy. We have

$$E = \frac{1}{2}Mv^2 + Mgl = 2\cdot\frac{1}{2}kx_m + Mg(l - x_m) \tag{91}$$

    Using $v = \sqrt{2gh}$ gives

$$kx_m^2 = Mg(x_m + h) \tag{92}$$

  iii At this instant the elastic force on spring upward must be equal to the weight of A, so

$$Mg = 2kx_e \tag{93}$$

    and thus

$$x_e = \frac{Mg}{2k} \tag{94}$$

  iv Total energy satisfies

$$E = 2\cdot\frac{1}{2}kx_e^2 + Mg(l + x_e) \tag{95}$$

    It also equals to

$$E = \frac{1}{2}Mv^2 + Mgl = Mg(h + l) \tag{96}$$

Equating the above two, and use the previous result $x_e = \frac{Mg}{2k}$ gives

$$h = x_e + \frac{kx_e^2}{Mg} = \frac{3Mg}{4k} \tag{97}$$

$\square$

Problem 4.

Solutions.   (a)

    i The charge is distributed on the surface of the sheet.
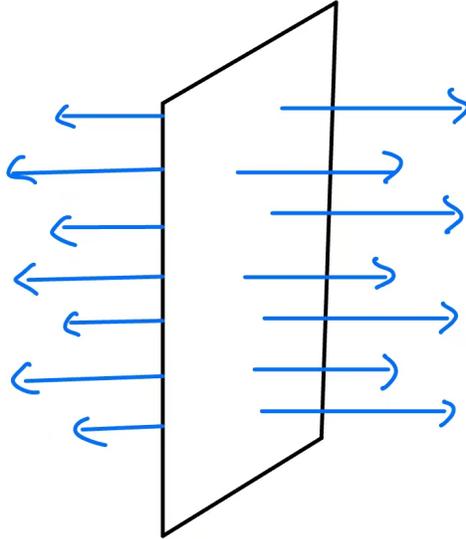
    ii As shown in the figure

Figure 9: 4.(a).ii

    Field lines are directing outwards from the surface, and perpendicular to the surface.

    iii Denser field lines represent stronger electric field, whereas less dense field lines represent weaker electric field.

    iv The force experienced by the point charge is

$$F = qE_s \tag{98}$$

(b)

i(i) The charge density of the sphere is

$$\sigma = \frac{Q}{4\pi r^2} \tag{99}$$

Consider spherical Gaussian surface of radius extends just outside of r,

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \tag{100}$$

i(ii) For this sphere,

$$E = \frac{\sigma}{\varepsilon_0} \tag{101}$$

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i(iii) Consider a Gaussian surface very tightly encloses the flat surface of area A, by symmetry and Gauss's Law,

$$2AE_s = \frac{Q}{\varepsilon_0} \tag{102}$$

Since $\sigma = \frac{Q}{A}$, we have

$$E_s = \frac{\sigma}{2\varepsilon_0} \tag{103}$$

ii(i) According to previous calculations.

$$E_{\text{sphere}} = \frac{\sigma}{\varepsilon_0} \tag{104}$$

and

$$E_s = \frac{\sigma}{2\varepsilon_0} \tag{105}$$

So we have

$$E_s = \frac{1}{2}E_{\text{sphere}} \tag{106}$$

ii(ii) When the field E' in the hole just inside the sphere due to the charge on the rest of the sphere is vectorially summed with the electric field due to the inside surface δA gives 0. So they have the same magnitude.

$$E' = E_s = \frac{\sigma}{2\varepsilon_0} \tag{107}$$

ii(iii) The force

$$F = E'\sigma\delta A = \frac{\sigma^2}{2\varepsilon_0}\delta A \tag{108}$$

Since $\sigma = \varepsilon_0 E_{\text{sphere}}$ we have

$$F = \frac{1}{2}\varepsilon_0 E_{\text{sphere}}^2 \delta A \tag{109}$$

ii(iv) The pressure

$$P = \frac{F}{\delta A} = \frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0 E_{\text{sphere}}^2 \tag{110}$$

(c)

i Maximum potential

$$V = \frac{Q}{4\pi\varepsilon_0 r} \tag{111}$$

Maximum electric field

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \tag{112}$$

So

$$r = \frac{V}{E} = 2.3 \times 10^{-3}\,\text{m} \tag{113}$$

ii With this same radius, the charge

$$Q = 4\pi\varepsilon_0 rV = 1.8 \times 10^{-9}\,\text{C} \tag{114}$$

iii The ground has potential 0 and the tube has potential -7000 V, so the potential difference from tube to the ground is $\Delta V = 7000$ V. Since this potential is negative of the result of the previous question, and the water formed at the tube needs to have the same potential, the charge on the surface of the water drop has to be negative of the previous result such that $q = -Q = -1.8 \times 10^{-9}$ C. Water has density $\rho = 1.0 \times 10^3$ kg/m^3. The radius of the rain drop is the same as previous questions. By energy conservation

$$\frac{1}{2}mv^2 - mgh + q\Delta V = 0 \tag{115}$$

where

$$m = \frac{4}{3}\pi r^3 \rho \tag{116}$$

is the mass of the drop. Solve gives

$$v = \sqrt{2gh - \frac{3q\Delta V}{2\pi r^3 \rho}} = 1.56\,\text{m/s} \tag{117}$$

iv The current

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\mathrm{d}Q}{\mathrm{d}V}\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{|q|}{\frac{4}{3}\pi r^3}\frac{\mathrm{d}V}{\mathrm{d}t} \tag{118}$$

where V is the volume of water follow and the absolute sign is because we want the magnitude of the current, and from the question description we have $\frac{\mathrm{d}V}{\mathrm{d}t} = 64/(60 \times 60) \times 10^{-6}\,\text{m}^3/\text{s}$ Substitute in gives $I = 6.0 \times 10^{-10}$ A.

(d)

i As shown in the figure By Snell's Law and basic plane geometry,

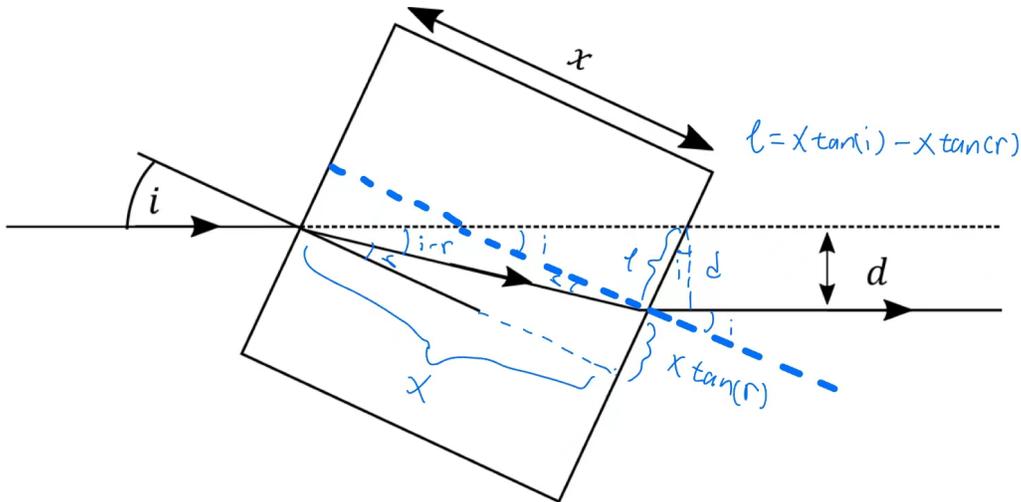

Figure 10: 4.(d).i

$$\begin{aligned}
\sin(i) &= n\sin(r) \\
d &= l\cos(i) \\
l &= x(\tan(i) - \tan(r))
\end{aligned} \tag{119}$$

By small angle approximation,

$$\begin{aligned}
\sin\theta &\approx \tan\theta \approx \theta \\
\cos\theta &\approx 1
\end{aligned} \tag{120}$$

Apply this to the above equations gives

$$d \approx \frac{n-1}{n}xi \tag{121}$$

ii We have

$$\dot{d} = \frac{n-1}{n}x\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{n-1}{n}x\omega = \frac{n-1}{n}x\cdot 2\pi f = 21\,\mathrm{m/s} \tag{122}$$

$\square$

Problem 5.

Problem 6.