BPhO Round 1

Section 1

15th November 2018

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This question paper must not be taken out of the exam room

Instructions

Time: **1 hour 20 minutes** for this section.

Questions: Students may attempt any parts of *Section 1*, but are not expected to complete all parts.

Working: Working, calculations, explanations and diagrams, properly laid out, must be shown for full credit. The final answer alone is not sufficient. Writing must be clear.

Marks: A **maximum of 50 marks** can be awarded for *Section 1*. There is a total of **87** marks allocated to the problems of Question 1 which makes up the whole of *Section 1*.

Instructions: You are allowed any standard exam board data/formula sheet.

Calculators: Any standard calculator may be used, but calculators cannot be programmable and must not have symbolic algebra capability.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets. Graph paper and formula sheets should also be made available. Students should ensure that their **name** and their **school/college** are clearly written on each and every answer sheet. Number each question clearly.

Setting the paper: There are two options for sitting BPhO Round 1:

a. *Section 1* and *Section 2* may be sat in one session of 2 hours 40 minutes plus 5 minutes reading time (for *Section 2*). *Section 1* should be collected in after 1 hour 20 minutes and then *Section 2* given out.

b. *Section 1* and *Section 2* may be sat in two sessions on separate occasions, with 1 hour 20 minutes plus 5 minutes reading time allocated for *Section 2*. If the paper is taken in two sessions on separate occasions, *Section 1* must be collected in after the first session and *Section 2* handed out at the beginning of the second session.

Important Constants

Constant	**Symbol**	**Value**
Speed of light in free space	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Elementary charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
Planck constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Mass of electron	m_{e}	$9.11 \times 10^{-31}\,\mathrm{kg}$
Mass of proton	m_{p}	$1.67 \times 10^{-27}\,\mathrm{kg}$
atomic mass unit	u	$1.661 \times 10^{27}\,\mathrm{kg}$
(1u is equivalent to 931.5 MeV)		
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^{3}\,kg^{-1}\,s^{-2}}$
Acceleration of free fall at Earth's surface	g	$9.81\,\mathrm{m\,s^{-2}}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Avogadro constant	N_{A}	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Mass of Sun	M_{S}	$1.99 \times 10^{30}\,\mathrm{kg}$
Radius of Earth	R_{E}	$6.37 \times 10^{6}\,\mathrm{m}$

$$T_{(\mathrm{K})} = T_{(^{\circ}\mathrm{C})} + 273$$

Volume of a sphere $= \frac{4}{3}\pi r^3$

$$e^x \approx 1 + x + \ldots \qquad x \ll 1$$

$$(1+x)^n \approx 1 + nx \qquad x \ll 1$$

$$\frac{1}{(1+x)^n} \approx 1 - nx \qquad x \ll 1$$

$$\sin\theta \approx \theta \qquad \text{for } \theta \ll 1;$$

$$\tan\theta \approx \theta \qquad \text{for } \theta \ll 1;$$

$$\cos\theta \approx 1 - \theta^2/2 \qquad \text{for } \theta \ll 1;$$

Question 1

a) The Milky Way galaxy has a period of rotation of 240×10^6 years. The Sun is $26\,000$ light years from the centre of the galaxy. How fast is the Sun moving with respect to the centre of the galaxy, given in units of $\mathrm{m\,s^{-1}}$?
A light year is the distance that light travels in one year of 365.25 days.

[3]

$$V = \omega r = \frac{2\pi}{T} r = \frac{2\pi \cdot 26000 \cdot 365.25 \cdot 3 \cdot 10^8}{240 \cdot 10^6 \cdot 365}$$

$$= 2.04 \times 10^4 \ m/s$$

b) A smooth sphere of radius $6.0\,\mathrm{cm}$ is suspended from a thread of length $9.0\,\mathrm{cm}$ attached to a smooth wall as shown in shown in **Figure 1**. If the mass of the sphere is $0.5\,\mathrm{kg}$, calculate the tension, T, in the thread.

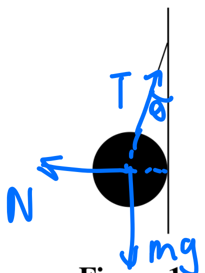
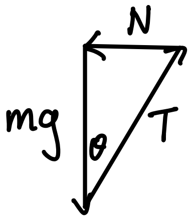


Figure 1

$$\sin\theta = \frac{r}{r+l} = \frac{6}{9+6} = \frac{6}{15} = \frac{2}{5}$$

$$\cos\theta = \sqrt{1-\left(\frac{2}{5}\right)^2} = \frac{\sqrt{21}}{5}$$



$$mg = T\cos\theta$$

$$\therefore T = \frac{5}{\sqrt{21}} mg = \frac{5}{\sqrt{21}} \cdot 0.5 \cdot 9.8 = 5.35 N$$

c) The displacement of an object is determined by the following function:

$$s = 2t^3 - 9t^2 + 12t + 4$$

where s is the displacement in metres, and t the time elapsed in seconds. Determine

(i) the times when the object comes to rest,
(ii) the time when the acceleration is zero,
(iii) the object's velocity when its acceleration is zero,
(iv) the object's accelerations when its velocity is zero.

[4]

(i) $V = \frac{ds}{dt} = 6t^2 - 18t + 12 = 6(t-1)(t-2)$ $\therefore V = 0$ when $t = 1, 2$ s

(ii) $a = \frac{dv}{dt} = 12t - 18 = 6(2t-3)$ $\therefore$ $a = 0$ when $t = 1.5$ s

(iii) At $t = 1.5$, $V = 6(0.5)(-0.5) = -1.5 \ m/s$

(iv) At $t = 1$, $a = -6 \ m/s^2$

$t = 2$, $a = 6 \ m/s^2$

d) The distance in which a train can be stopped is given by:

$$s = av + bv^2$$

where s is the stopping distance, v the initial velocity, and a and b are constants. When moving at $40\,\mathrm{km\,hr^{-1}}$, the train can be stopped in $100\,\mathrm{m}$, and at $80\,\mathrm{km\,hr^{-1}}$ it can be stopped in $280\,\mathrm{m}$.

Find the greatest speed such that the train can be stopped in $500\,\mathrm{m}$.

[4]

$$\begin{cases} 100 = a\cdot 40 + b\cdot 40^2 \\ 280 = a\cdot 80 + b\cdot 80^2 \end{cases} \Rightarrow \begin{cases} a = \frac{3}{2} \\ b = \frac{1}{40} \end{cases}$$

$$S = \frac{3}{2}v + \frac{1}{40}v^2$$

when $500 = \frac{3}{2}v + \frac{1}{40}v^2 \rightarrow V = 115\,\mathrm{km/hr}$

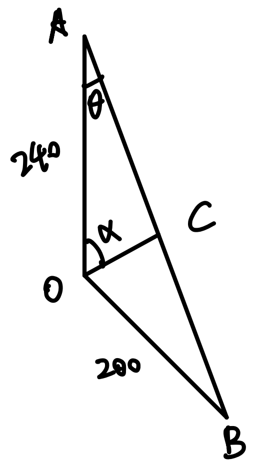
e) Two planes set out at the same time from an aerodrome. The first flies north at $360\,\mathrm{km\,h^{-1}}$, the second south-east at $300\,\mathrm{km\,h^{-1}}$. After 40 minutes they both turn and fly towards each other. Calculate

(i) the bearing, and

(ii) the distance

of the meeting point from the aerodrome.

[7]



$\angle AOB = 135°$

$AB = (240^2 + 200^2 - 2\cdot 240\cdot 200\cdot \cos(135°))^{\frac{1}{2}}$

$= 406.8\,\mathrm{km}$

$AC = \frac{360}{360+300}\cdot AB = 221.9\,\mathrm{km}$

In $\triangle AOB$: $\frac{AB}{\sin(135°)} = \frac{OB}{\sin\theta} \rightarrow \theta = 20.34°$

In $\triangle AOC$: $OC = (240^2 + 221.9^2 - 2\cdot 240\cdot 221.9\cdot \cos(20.34°))^{\frac{1}{2}}$

$= 83.5\,\mathrm{km}$

$\frac{OC}{\sin\theta} = \frac{AC}{\sin\alpha} \rightarrow \alpha = 67.5°$

f) A neutron moving through heavy water strikes an isolated and stationary deuteron (the nucleus of an isotope of hydrogen) head-on in an elastic collision.

m 2m

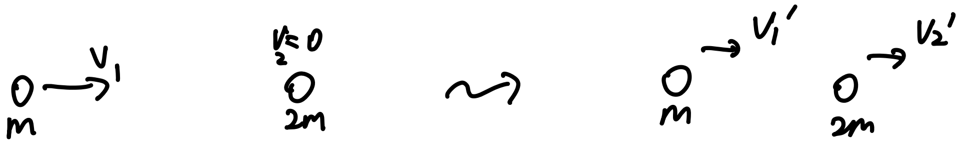
(i) Assuming the mass of the neutron is equal to half that of the deuteron, find the ratio of the final speed of the deuteron to the initial speed of the neutron.

(ii) What percentage of the initial kinetic energy is transferred to the deuteron?

(iii) How many such collisions would be needed to slow the neutron down from $10\,\mathrm{MeV}$ to $0.01\,\mathrm{eV}$?

[6]

(i)



$$\begin{cases} m v_1 = m v_1' + 2m v_2' & \Leftrightarrow \frac{d}{dt}\Sigma P = 0 \\ v_1 - 0 = -(v_1' - v_2') & \Leftrightarrow \text{elastic} \end{cases}$$

$$\therefore \quad -v_1' = 2v_2' - v_1 = v_1 - v_2' \Rightarrow 3v_2' = 2v_1 \Rightarrow \underline{\frac{v_2'}{v_1} = \frac{2}{3}}$$

(ii)

$$v_1' = \frac{2}{3}v_1 - v_1 = -\frac{1}{3}v_1$$

$$K_D = \frac{1}{2}(2m)\left(\frac{2}{3}v_1\right)^2 = \frac{1}{2}mv_1^2 \cdot \frac{8}{9}$$

$$K_i = \frac{1}{2}mv_1^2$$

$$\Rightarrow \frac{K_D}{K_i} = \frac{8}{9} = 88.9\%$$

(iii)

$$\left(1 - \frac{8}{9}\right)^N = \frac{0.01}{10 \times 10^6} \Longrightarrow N = 9.43$$

$\therefore$ Need $\underline{10}$ Collisions.

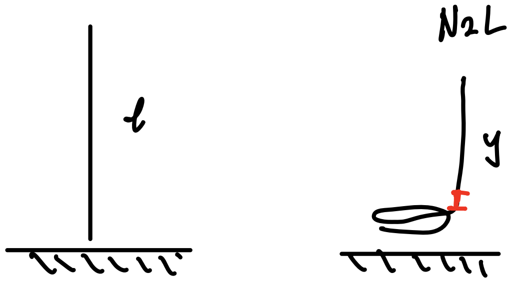
g) A uniform chain of mass per unit length, μ, is suspended from one end above a table, with the lower end just touching the surface. The chain is released, falls and comes to rest on the table without bouncing.

(i) Determine an expression, in terms of μ and the gravitational field strength g, for the reaction force exerted by the table on the chain as a function of time, t.
Hint: you might consider F in the form $F = \frac{\Delta m}{\Delta t}v$.

(ii) In terms of the total weight W of the chain, what is the maximum reaction force exerted by the table, and at what time during the fall does this occur?

[6]

(i)



N2L : $F = \frac{dp}{dt}$

$$\rightarrow N - \mu l g = \frac{d}{dt}(\mu y \dot{y})$$

$\left(\begin{array}{c} \dot{y} < 0 \\ p < 0 \\ \uparrow \text{ is } + \end{array} \right)$

$$\therefore N = \mu l g + \mu \dot{y}^2 + \mu y \ddot{y}$$

Ideal chain not stretched $\rightarrow$ No tension $\rightarrow$ $\ddot{y} = -g$, $\dot{y} = -\sqrt{2g(l-y)}$
(free fall)

$$\therefore \quad N = \mu l g + 2\mu g(l-y) - \mu y g = 3\mu g(l-y)$$

and $l - y = \frac{1}{2}gt^2$ (height fallen) $\therefore$ $\underline{N = \frac{3}{2}\mu g^2 t^2}$

(ii) As $y = 0$, $l = \frac{1}{2}gt^2$, At the end of the fall, $N_{max} = 3\mu g l = \underline{3W}$
$(W = \mu g L)$

h) A small particle of mass m can slide without friction round the inside of a cylindrical hole of radius r, in a rectangular shaped object of mass M. The rectangular object is held between rigid walls by small wheels so that it can slide up and down without friction, as shown in **Figure 2**. If the small particle m is initially at rest at the bottom of the cylindrical hole, and is then given an impulse to give it a speed v, what is the minimum speed v needed to just lift the rectangular mass M off the ground?

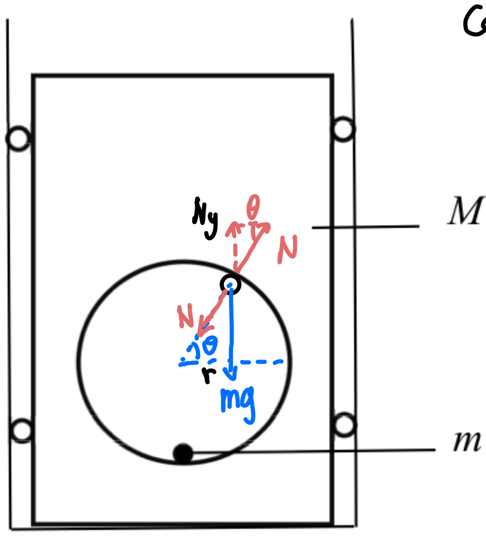


Figure 2

Condition to just lift off : $\underline{N_y = N\sin\theta = Mg}$ ①

$$\begin{cases} \text{E conserve : } \frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + mgr(1+\sin\theta) & ② \\ \text{Circular motion : } N + mg\sin\theta = m\frac{v'^2}{r} & ③ \end{cases}$$

② $\Rightarrow$ $v'^2 = v^2 - 2gr(1+\sin\theta)$

Define $\boxed{\begin{cases} v^2 = kgr \\ b = \frac{M}{m} \end{cases}}$ for constant $k > 0$
" " $b > 0$

② $\Rightarrow$ $v'^2 = (k-2)gr - 2\sin\theta\, gr$ ④

④ → ③ $\Rightarrow$ $N = [(k-2) - 3\sin\theta]mg$ ⑥

⑥ → ① $\Rightarrow$ $N_y = [(k-2)\sin\theta - 3\sin^2\theta]mg = Mg$

$\Rightarrow$ $\boxed{3x^2 - (k-2)x + b = 0, \quad \text{for } x = \sin\theta}$ ⑦

we want a minimum $k = \frac{v^2}{gr}$ such that ⑦ holds.

for ⑦ to have real solution : discriminant $\Delta = b^2 - 4ac \geq 0$

$\therefore$ $(k-2)^2 - 12b \geq 0$ $\therefore$ $\underline{k \geq 2 + \sqrt{12b}}$

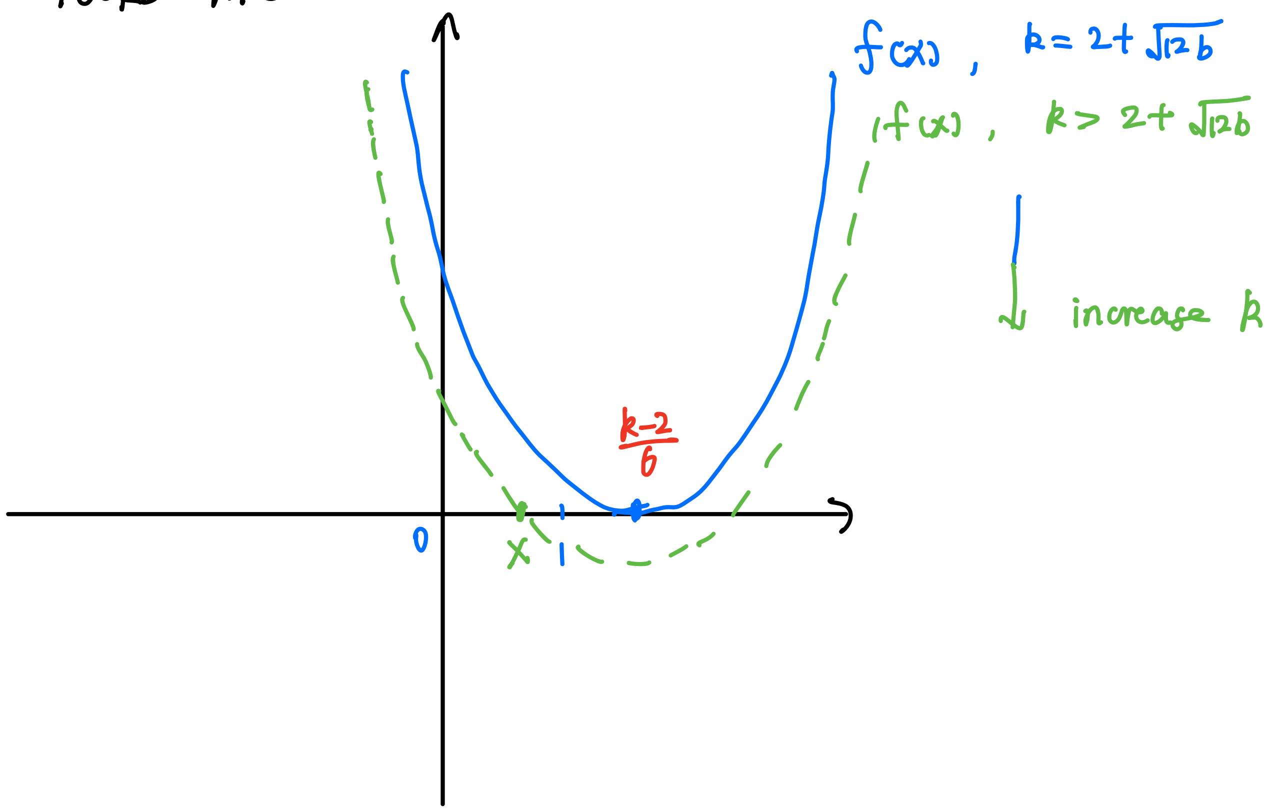
$f(x) = 3(x - \frac{k-2}{6})^2 - [\frac{(k-2)^2}{12} - b] = 0$ ⑦

So when $k = 2 + \sqrt{12b}$, $x = \frac{k-2}{6}$

$\because$ $x = \sin\theta$ $\therefore$ $x \leq 1$, so although $k \geq 2 + \sqrt{12b}$ always hold, we still need $x = \frac{k-2}{6} \leq 1$ $\Rightarrow$ $\underline{k \leq 8}$, otherwise k cannot take this lowest value.

Solve $2 + \sqrt{12b} \leq 8$ $\Rightarrow$ $\underline{b \leq 3}$, so when $b \leq 3$, k can take $k_{min} = 2 + \sqrt{12b}$.

If $b > 3$, when $k = 2 + \sqrt{12b}$, $x = \frac{k-2}{6} > 1$, It's graph looks like



For fixed b, in order to get a solution of x such that $x \leq 1$, we need to increase k from $2+\sqrt{12b}$ to a value higher.

From graph, as we increase k, the first solution of $x \leq 1$ it reaches is $x = 1 \Rightarrow \sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$, at the top of circle.

From ⑦, when $x=1$, $3-(k-2)+b=0 \Rightarrow k = 5+b$

So overall, condition to just lift off:

$$\boxed{\begin{aligned} &\text{when } b=\frac{M}{m} \leq 3, \quad k = 2+\sqrt{12b} \Rightarrow v = \sqrt{\left(2+\sqrt{\frac{12M}{m}}\right)gr} \\ &\text{when } b=\frac{M}{m} > 3, \quad k = 5+b \Rightarrow v = \sqrt{\left(5+\frac{M}{m}\right)gr} \end{aligned}}$$

It can be easily verified that $(5+b)-(2+\sqrt{12b}) = (\sqrt{b}-\sqrt{3})^2 \geq 0$ and achieves 0 precisely at $b=3$

i) Two resistors and two cells are connected in the circuit shown in **Figure 3**. One cell has an e.m.f. of 2.0 V and an internal resistance of 1.0 Ω, the other an e.m.f. of 1.5 V and an internal resistance of 0.5 Ω. The resistors are connected in series and the point between them is at earth, i.e. zero potential. Calculate

(i) the current through the cells,

(ii) the potential difference across each cell, and

(iii) the potential, relative to earth, at points A and B.

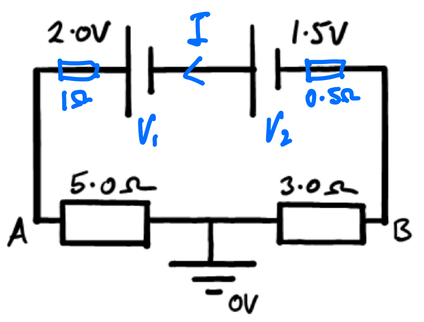


Figure 3

(i) $I = \dfrac{2+1.5}{1+0.5+5+3} = 0.368\,A$

(ii)

$V_1 = \varepsilon_1 - Ir_1 = 2.0 - 0.368 \cdot 1 = 1.63\,V$

$V_2 = \varepsilon_2 - Ir_2 = 1.5 - 0.368 \cdot 0.5 = 1.32\,V$

(iii) $V_A - 0 = 0.368 \cdot 5.0 \Rightarrow V_A = 1.84\,V$

$0 - V_B = 0.368 \cdot 3.0 \Rightarrow V_B = 1.10\,V$

j) A thick-bottomed, cylindrical glass beaker is placed on a bench. Water and oil are poured into the beaker and form discrete layers, as shown in **Figure 4**. The bottom of the beaker is 1.8cm thick, the water is 1.2 cm deep, and the oil layer is 0.8 cm deep.

(i) Draw a diagram showing the path of a ray at a small angle to the normal, travelling from the underside of the beaker and being refracted through the layers.

(ii) Assuming the angles of deviation of the ray are small, calculate the apparent vertical displacement of the lab bench when viewed from above.
The refractive indices are 1.5, 1.3 and 1.1 for the glass, water and oil respectively.

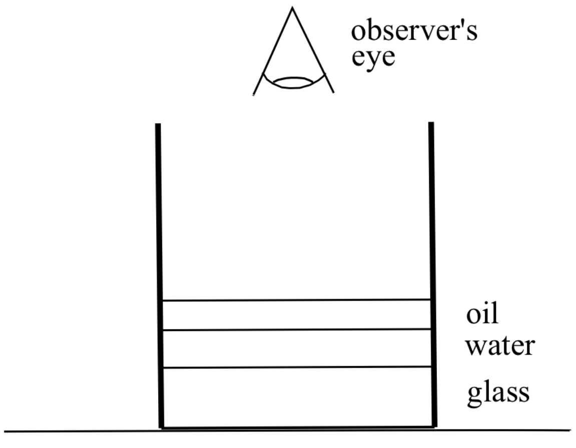
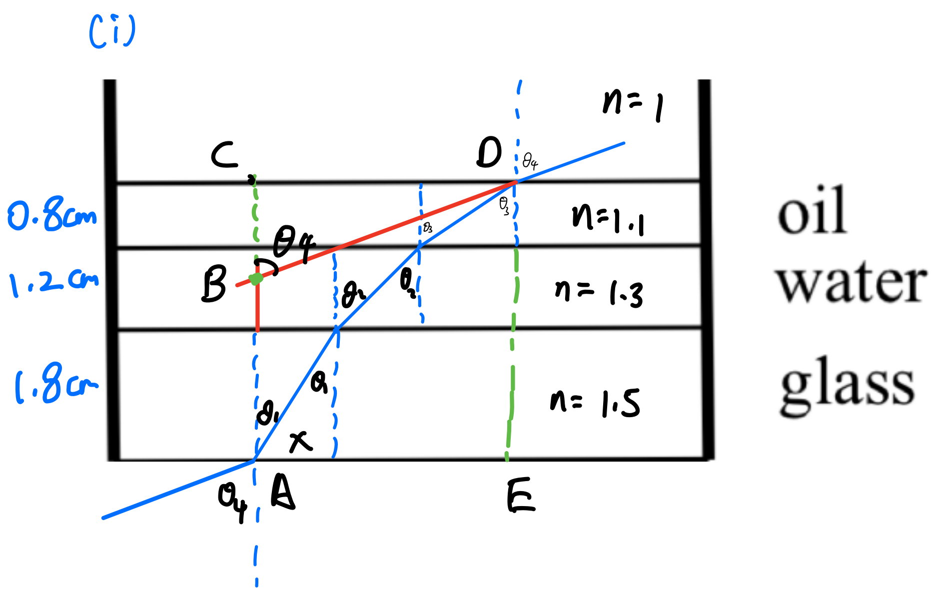


Figure 4

(i)



(ii) $1.5\sin\theta_1 = 1.3\sin\theta_2 = 1.1\sin\theta_3 = \sin\theta_4$

$CD = AE = 1.8\tan\theta_1 + 1.2\tan\theta_2 + 0.8\tan\theta_3$

$CD = BC\tan\theta_4$

small angle $\sin\theta \simeq \tan\theta$ $\therefore$ $CD = 1.8\sin\theta_1 + 1.2\left(\frac{1.5}{1.3}\right)\sin\theta_1$

$+ 0.8\left(\frac{1.5}{1.1}\right)\sin\theta_1 = 4.28\sin\theta_1$

$CD = BC \sin\theta_4 = 1.5\, BC \sin\theta_1$

$\Rightarrow \quad 1.5\, BC = 4.28 \quad \Rightarrow \quad BC = 2.85 \text{ cm}$

Apparent displacement $= AB = AC - BC$

$= 1.8 + 1.2 + 0.8 - 2.85$

$= 0.95 \text{ cm}$

k) A person might reasonably expect to jump a height of 1 m on Earth. On a planet with a density two thirds that of Earth, and radius twice that of the Earth, to what height might the person jump? Assume that they supply the same energy to make the jump

[4]

Gravitational field strength $g = \frac{GM}{r^2} = \frac{G}{r^2}\left(\frac{4}{3}\pi r^3 \rho\right)$

$\rightarrow \quad g = \left(\frac{4\pi G}{3}\right)\rho r$

$\because \quad \rho' = \frac{2}{3}\rho, \quad r' = 2r \qquad \therefore \; g' = \frac{4}{3} g$

same energy $\quad mgh = mg'h'$

$\therefore \; h' = \frac{3}{4} h = \frac{3}{4} = \underline{0.75 \text{ m}}$

l) A pond containing water of density ρ is covered to a depth b by oil of density $\frac{2}{3}\rho$. A long wood block of square cross section $4b \times 4b$, with the same density as the oil, floats in the pond, as shown in **Figure 5**. What fraction of the wood block is immersed below the top of the surface oil level?

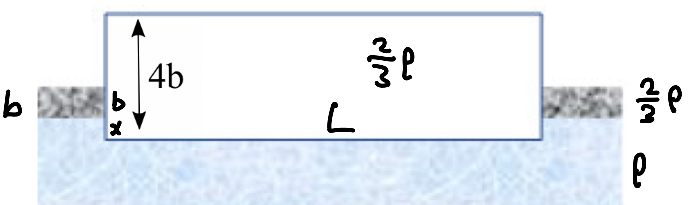


Figure 5

$G = U$

$\left(\frac{2}{3}\rho\right) g (4b)^{\cancel{2}} \cancel{L}$

$= \left(\frac{2}{3}\rho\right) \cancel{g} (\cancel{4b})(b)\cancel{L} + \rho \cancel{g} (\cancel{4b})(x)\cancel{L}$

$\rightarrow \quad x = \left(\frac{2}{3}\right)(4b) - \frac{2}{3} b = \frac{2}{3} \cdot 3b = 2b$

$x + b = 3b, \quad$ fraction immersed $= \frac{3b}{4b} = \frac{3}{4}$

m) A volume of $80\,\mathrm{cm^3}$ of water in a copper calorimeter of mass $150\,\mathrm{g}$ takes 12 minutes to cool from $40\,°\mathrm{C}$ to $15\,°\mathrm{C}$ in a cold room. The same volume of ethanol of density $0.8\,\mathrm{g\,cm^{-3}}$ takes 8 minutes to cool also from $40\,°\mathrm{C}$ to $15\,°\mathrm{C}$ in the same calorimeter in the same circumstances. Calculate the specific heat capacity of ethanol.
The specific heat capacity of copper = $400\,\mathrm{J\,kg^{-1}\,°C^{-1}}$ and of water = $4200\,\mathrm{J\,kg^{-1}\,°C^{-1}}$.
The density of water, $\rho_w = 1.0\,\mathrm{g\,cm^{-3}}$.

[5]

we assume that the rate of loss of thermal energy is the same in both cases at any given temperature.

$$\Delta Q_1 = (C_w \rho_w V_w + C_c M_c)\Delta T$$
$$= (4200 \cdot 10^3 \cdot 80 \cdot 10^{-6} + 400 \cdot 0.15)(40 - 15)$$
$$= 9900\,\mathrm{J}$$

$$\Delta Q_2 = (C_e \rho_e V_e + C_c M_c)\Delta T$$
$$= (C_e \cdot 0.8 \cdot 10^3 \cdot 80 \cdot 10^{-6} + 400 \cdot 0.15)(40 - 15)$$
$$= 1.6\,C_e + 1500$$

$$\because \frac{\Delta Q_1}{\Delta t_1} = \frac{\Delta Q_2}{\Delta t_2} \quad \therefore \quad \frac{9900}{12} = \frac{1.6\,C_e + 1500}{8}$$

$$\rightarrow \quad \underline{C_e = 3187.5\ \mathrm{J\,kg^{-1}\,°C^{-1}}}$$

n) A steel girder is planted securely between two sides of a ravine in order to provide a bridge. The total cross-sectional area of the girder is $30\,\mathrm{cm^2}$, and the length of the girder is $4.0\,\mathrm{m}$. Installed at a temperature of $5\,°\mathrm{C}$, the temperature now rises to $20\,°\mathrm{C}$. Calculate the force exerted by the girder due to the change in temperature, assuming the ends do not move.
Young modulus of steel = $2.0 \times 10^{11}\,\mathrm{Pa}$
Linear expansivity of steel (fractional expansion per unit temperature rise) = $1.2 \times 10^{-7}\,°\mathrm{C}^{-1}$ at $5\,°\mathrm{C}$.

[4]

$$E = \frac{Fl}{Ae} \rightarrow F = \frac{EA\Delta l}{l} \quad (e = \Delta l) \quad \Delta l = \alpha \Delta T l$$

$$\therefore F = EA\alpha\Delta T \quad (\because \sqrt{A} \ll l \ \therefore \text{ assume } A \text{ does not change})$$

$$F = (2 \cdot 10^{11})(30 \times 10^{-4})(1.2 \cdot 10^{-7})(20 - 5) = 1080\,\mathrm{N}$$

o) A narrow beam of monochromatic light falls on a diffraction grating of 1200 lines mm^{-1}, and two diffracted beams of successive orders are observed at 14° and 73° to the normal, both of them on the same side of the normal. The incident beam of light is not along the normal to the grating.

(i) Sketch a diagram to show the path difference between rays passing through adjacent slits, for a ray incident on the diffraction grating at angle θ_1, and for the corresponding ray emerging from the grating at angle θ_2, with respect to the normal.

(ii) Derive an equation relating the angles θ_1 and θ_2 to the order of diffraction, n, and the wavelength, λ.

Determine:

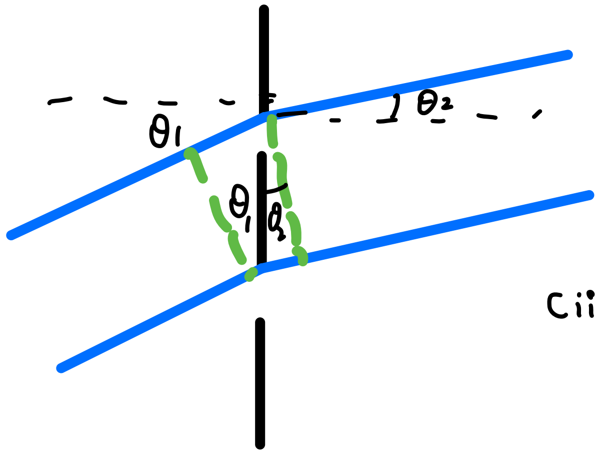
(iii) The wavelength of the light used.

(iv) The angle of incidence of the beam on the grating.

(v) The angle of diffraction of a third transmitted beam.

[6]

(i)



(ii) $d = \frac{1}{1200}\text{mm} = 8.33 \times 10^{-7}\text{m}$

$d(\sin\theta_1 - \sin\theta_2) = n\lambda$

(iii) $d(\sin\theta_1 - \sin 14^\circ) = (n+1)\lambda$

$d(\sin\theta_1 - \sin 73^\circ) = n\lambda$

$\Rightarrow \lambda = d(\sin 73^\circ - \sin 14^\circ)$

$\therefore \lambda = \underline{595\text{ nm}}$

(iv) $\sin\theta_1 = \sin 73^\circ + \frac{n\lambda}{d} = \sin 73^\circ + \frac{5}{7}n$

$\because -1 \leq \sin\theta_1 \leq 1 \quad \therefore$ allowed n is $n = 0, -1, -2$

$\sin\theta_1 = \sin 14^\circ + \frac{5}{7}(n+1)$ allowed $n+1 = 1, 0, -1$

$\therefore n = 0, -1, -2$

$\therefore \theta_1 = 14^\circ, 73^\circ, 28.2^\circ$ if not principal maximum

$\rightarrow \underline{\theta = 28.2^\circ}$

(v) $n = \pm 3$ not allowed in

$d(\sin 28.2^\circ - \sin\theta_2) = \pm 3\lambda$

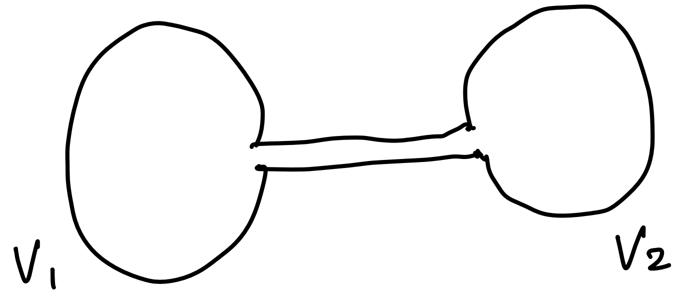
$\therefore$ No third beam.

p) Two identical spherical glass containers are joined by a narrow tube, whose volume is negligible compared to the spheres. The spheres contain air at 100 °C. One of the spheres is then heated by 50 °C whilst the other is cooled by 50 °C. This produces a small change in pressure, from P_{initial} to P_{final}, of the air in the system. What common temperature of the two spheres could produce the same final pressure P_{final}?

[4]

$PV = Nk_BT$

$T_0 = 373K = 100\,°C$



$$N_1 + N_2 = \frac{P_i V_1}{k_B T_0} + \frac{P_i V_2}{k_B T_0} = \frac{P_f V_1}{k_B (T_0 + \delta_0)} + \frac{P_f V_2}{k_B (T_0 - \delta_0)}$$

$$= \frac{P_f V_1}{k_B T_f} + \frac{P_f V_2}{k_B T_f}$$

$$V_1 = V_2 \Rightarrow \frac{P_f}{423k} + \frac{P_f}{323K} = \frac{2P_f}{T_f}$$

$$\therefore T_f \approx 366.3\,K = \underline{93.3\,°C}$$

q) A simple pendulum consists of a small mass on the end of a light, inextensible string, as shown in **Figure 6**. It swings from an initial angle $\theta = 14°$, for which it would have a period T_0, but it hits a wall elastically, which is at angle $\phi = 7°$ to the vertical. What is the new period of oscillation in terms of T_0?

(θ, ϕ are small angles such that $\sin\theta \approx \theta$ and $\sin\phi \approx \phi$).

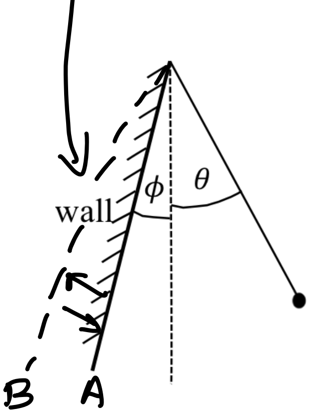


Figure 6

$\theta = \theta_0 \cos\omega t \quad , \quad \omega = \frac{2\pi}{T_0}$

$7° = 14° \cos\omega t$

$\omega t = \arccos(\frac{1}{2}) = \frac{\pi}{3}$

$\therefore t = \frac{\pi}{3} \cdot \frac{T_0}{2\pi} = \frac{T_0}{6}$,

$T = 2t = \frac{T_0}{3}$, T is the time taken from A to B then back to A

New period $T_0' = T_0 - T = T_0 - \frac{T_0}{3} = \underline{\frac{2}{3} T_0}$

r) Four charges are placed at the corners of a square of side 10 cm, as shown in **Figure 7**.

$A = +10 \times 10^{-9}\,\mathrm{C}$ $\quad a = 0.1\,\mathrm{m}$
$B = +8 \times 10^{-9}\,\mathrm{C}$
$C = -12 \times 10^{-9}\,\mathrm{C}$
$D = -6 \times 10^{-9}\,\mathrm{C}$

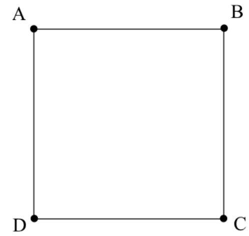
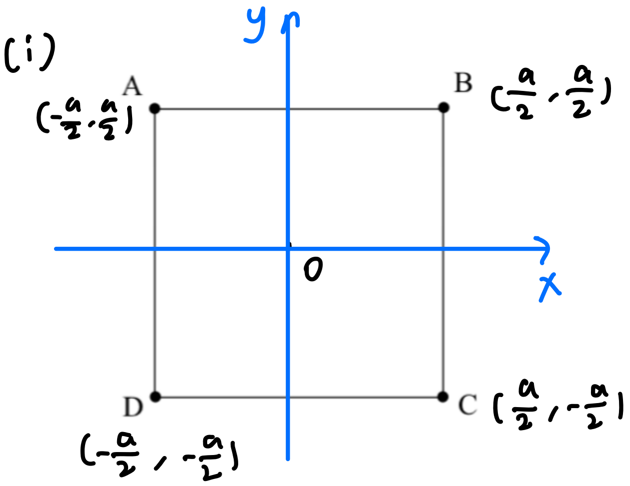


Figure 7

(i) Calculate the magnitude and direction of the electric field strength at the centre of the square.

(ii) Calculate the work done taking an electron from the centre to the mid-point of side CD.

(i)



$\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$

$$\underline{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^3} \underline{r}$$

$$\underline{E_A} = \frac{1}{4\pi\varepsilon_0} \frac{Q_A}{a^3} (2\sqrt{2}) \left(\frac{a}{2}, -\frac{a}{2}\right)$$

$$\underline{E_B} = \frac{1}{4\pi\varepsilon_0} \frac{Q_B}{a^3} (2\sqrt{2}) \left(-\frac{a}{2}, -\frac{a}{2}\right)$$

$$\underline{E_C} = \frac{1}{4\pi\varepsilon_0} \frac{Q_C}{a^3} (2\sqrt{2}) \left(-\frac{a}{2}, \frac{a}{2}\right), \quad \underline{E_D} = \frac{1}{4\pi\varepsilon_0} \frac{Q_D}{a^3} (2\sqrt{2}) \left(\frac{a}{2}, \frac{a}{2}\right)$$

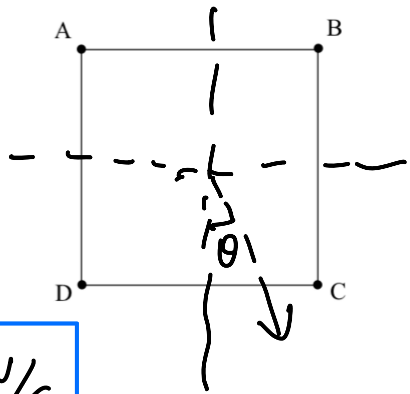
$$\underline{E_O} = \underline{E_A} + \underline{E_B} + \underline{E_C} + \underline{E_D}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{2\sqrt{2}}{a^3} \cdot \frac{a}{2} \left(Q_A - Q_B - Q_C + Q_D, -Q_A - Q_B + Q_C + Q_D\right)$$

$$= (8.99 \times 10^9) \cdot \frac{\sqrt{2}}{0.1^2} \times 10^{-9} \left(10 - 8 + 12 - 6, -10 - 8 - 12 - 6\right)$$

$$= 1271 \times (8, -36)$$

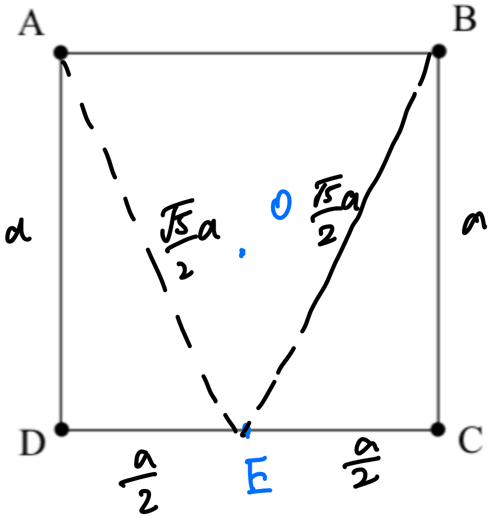
$$= (10168, -45756)\ \mathrm{N/C}$$



$$\therefore E = \sqrt{E_x^2 + E_y^2} = \boxed{4.69 \times 10^4\ \mathrm{N/C}}$$

$$\tan\theta = \frac{8}{36} \rightarrow \boxed{\theta = 12.5^\circ}$$

(ii)



$$V_O = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a}(Q_A+Q_B+Q_C+Q_D)$$

$$= 8.99 \cdot \frac{\sqrt{2}}{0.1}(10+8-12-6)$$

$$= 0$$

$$V_E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_A \cdot 2}{a\cdot\sqrt{5}} + \frac{Q_B\cdot 2}{a\cdot\sqrt{5}} + \frac{2Q_C}{a} + \frac{2Q_D}{a}\right]$$

$$V_E = 8.99 \cdot \frac{1}{0.1}\cdot\left[\frac{2Q_A}{\sqrt{5}} + \frac{2Q_B}{\sqrt{5}} + 2Q_C + 2Q_D\right]$$

$$= -1789\ V$$

electron from O to E

$$W = -e(V_E - V_O) = -1.6\times10^{-19}(-1789)$$

$$= \underline{2.86\times10^{-16}\ J}$$

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