

$$-\frac{dU}{dx} = F$$

Mechanics II. Ziyan Li

1 (a)

$$W_{ab} = \int_{x_a}^{x_b} F(x) dx = \int_{x_a}^{x_b} m \frac{dv}{dt} dx = \int_{x_a}^{x_b} m \frac{dx}{dt} dv$$

$$= \int_{v_a}^{v_b} m v dv = m \int_{v_a}^{v_b} v dv = \frac{1}{2} m v_b^2 - \frac{1}{2} m v_a^2$$

$$= T(b) - T(a)$$

(b)

$$\Delta U = - \int_{x_0}^{x_{min}} F(x) dx$$

$$= - \int_{x_0}^{x_{min}} \frac{k}{x^2} dx$$

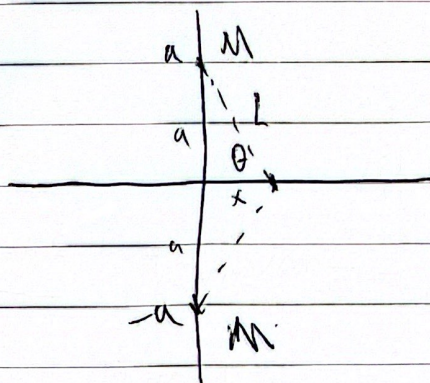
$$= - \left(-\frac{k}{x} \right) \Big|_{x_0}^{x_{min}} = \frac{k}{x_{min}} - \frac{k}{x_0}$$

$$\Delta U = -\Delta T \quad \therefore \frac{k}{x_{min}} - \frac{k}{x_0} = - \left(0 - \frac{1}{2} m u^2 \right)$$

$$\therefore \frac{k}{x_{min}} - \frac{k}{x_0} = \frac{1}{2} m u^2$$

$$\therefore x_{min} = \frac{k}{\frac{k}{x_0} + \frac{1}{2} m u^2} = \boxed{\frac{2 k x_0}{2 k + m u^2 x_0}}$$

2.



$$F(x) = - \frac{2 G M m}{L^2} \cos \theta$$

$$= - \frac{2 G M m}{x^2 + a^2} \left(\frac{x}{\sqrt{x^2 + a^2}} \right)$$

$$= - \frac{2 G M m x}{(x^2 + a^2)^{3/2}}$$

$$-\int_{h_0}^{h_1} -mg dh = mgh/h_0$$

$$- (x^2 + a^2)^{-1/2}$$

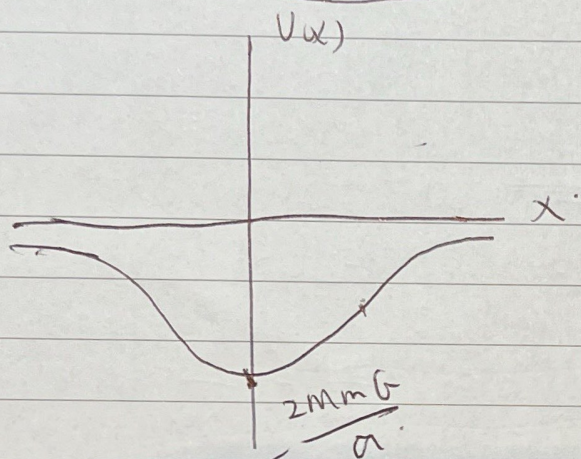
$$- (-\frac{1}{2})(x^2 + a^2)^{-1/2} (2x)$$

$$-\frac{1}{\sqrt{x^2 + a^2}} - \int -\frac{2mM G x}{(x^2 + a^2)^{3/2}} dx$$

$$U = -\int F(x) dx = -2mM G \int \frac{x}{(a^2 + x^2)^{3/2}} dx$$

$$= -2mM G \left(-1/\sqrt{x^2 + a^2} \right)$$

$$= -\frac{2mM G}{\sqrt{x^2 + a^2}}$$



$$F = -\frac{2mM G}{a^3} \frac{x}{(1 + \frac{x^2}{a^2})^{3/2}}$$

$$\sim -\frac{2mM G}{a^3} x \left(1 - \frac{3x^2}{2a^2} \right)^{-3/2}$$

$$= -\frac{2mM G}{a^3} x \left(1 - \frac{3x^2}{2a^2} \right)$$

$$\sim -\frac{2mM G}{a^3} x \quad [x \ll a]$$

$\therefore$ The possible motion is a simple harmonic motion when $x \ll a$.

$$(C) \quad T_0 + U_0 = T_1 + U_1$$

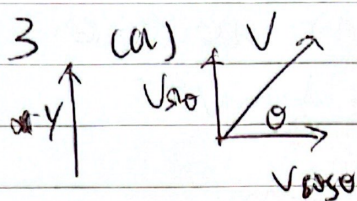
$$0 + \frac{2mM G}{\sqrt{a^2 + \frac{9}{4}a^2}} = \frac{1}{2} m V_{max}^2 + \frac{2mM G}{a}$$

$$\frac{2mM G}{\frac{5}{4}a} = \frac{1}{2} m V_{max}^2 - \frac{2mM G}{a}$$

$$\therefore \frac{1}{2} m V_{max}^2 = \frac{3}{5} \frac{mM G}{a}$$

$$\therefore V_{max} =$$

$$\sqrt{\frac{4GM}{5a}}$$



$$V \sin \theta t - \frac{1}{2} g t^2 = 0$$

$$t \neq 0, \quad t = \frac{2V \sin \theta}{g}$$

(b)

$$0^2 - V^2 = -2g y_{\max} \quad ; \quad 0^2 - V^2 \sin^2 \theta = -2g y_{\max}$$

$$y_{\max} = \frac{V^2 \sin^2 \theta}{2g}$$

(c)

$$R = V \cos \theta t = \frac{2V \sin \theta V \cos \theta}{g}$$

$$= \frac{V^2 \sin 2\theta}{g}$$

(d)

At maximum height!

$$U = mgh = \frac{mg V^2 \sin^2 \theta}{2g} = \frac{1}{2} m V^2 \sin^2 \theta$$

$$T = \frac{1}{2} m v^2 = \frac{1}{2} m V^2 \cos^2 \theta$$

$$U + T = \frac{1}{2} m V^2 (\sin^2 \theta + \cos^2 \theta) = \frac{1}{2} m V^2$$

(e)

	m	s	m	s
①	R = 972	t = 14.4	R' = 1057	t' = 20.78
②	R = 1160	t = 15.7	R' = 1285	t' = 22.99
③	R = 1349	t = 17.0	R' = 1516	t' = 24.88
④	R = 1539	t = 18.2	R' = 1762	t' = 26.82

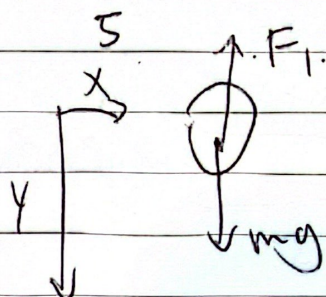
Air resistance may slow down the motion of the object.

Q 4. $\bar{F}$ = average air resistance.

$mg y - \bar{F} y = \frac{1}{2} m v^2$

$\therefore \bar{F} = \frac{mg y - \frac{1}{2} m v^2}{y} = \boxed{1.8 \text{ N}}$

The term average means the overall effect of this force over the given distance is the same as the actual forces, which varies with time.



$F_1 = mg$ at terminal v

$\therefore 6\pi\eta r v_t = mg$

$\therefore v_t = \frac{mg}{6\pi\eta r} = \boxed{0.2 \text{ m/s}}$

In $+y$ direction, $m \frac{dv}{dt} = mg - F_1 = mg - 6\pi\eta r v$

$\therefore \frac{dv}{dt} = g - \frac{6\pi\eta r}{m} v$ let $\lambda = \frac{6\pi\eta r}{m}$

$\Rightarrow \boxed{\frac{dv}{dt} = g - \lambda v}$

$\int \frac{dv}{g - \lambda v} = \int dt \quad \therefore -\frac{1}{\lambda} \ln(g - \lambda v) = t + C$

$\therefore \ln(g - \lambda v) = -\lambda t + C$

$\therefore g - \lambda v = C e^{-\lambda t}$ ($t=0 \rightarrow v=0 \Rightarrow C=g$)

$\therefore \lambda v = g(1 - e^{-\lambda t})$

$\therefore v = \frac{g(1 - e^{-\lambda t})}{\lambda}$

$$ax^2 + bx + c = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{c}{b}$$

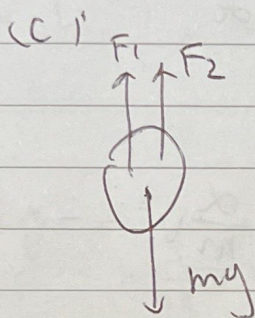
Solve for $\frac{g(1 - e^{-\lambda t})}{\lambda} = 0.95 \frac{g}{\lambda}$

gives $1 - e^{-\lambda t} = 0.95$

$\therefore e^{-\lambda t} = 0.05$

$\therefore -\lambda t = \ln(0.05)$

$\therefore t = \frac{-\ln(0.05)}{\lambda} = 31.2 \text{ s}$



$$m \frac{dv}{dt} = mg - F_1 - F_2$$

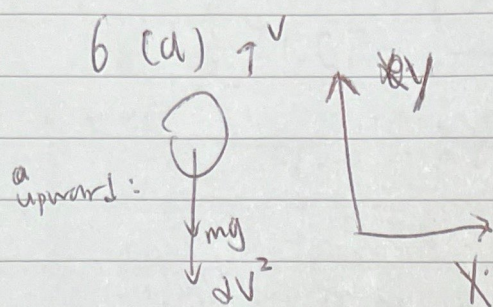
$$0 = mg - m\lambda v_t - 0.87a v_t^2$$

$$\therefore \cancel{0.87a} \quad 0.87a v_t^2 + m\lambda v_t - mg = 0$$

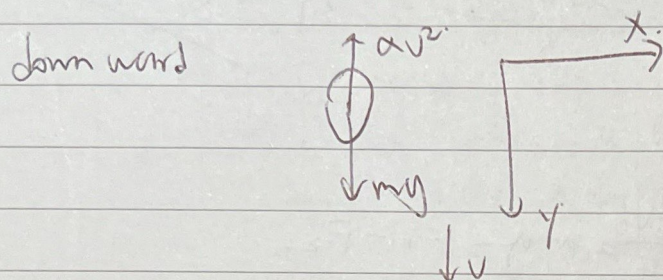
$$\cancel{v_t} \quad \text{or} \quad 0.1305 v_t^2 + 4.307 \times 10^{-5} v_t - 0.0049 = 0$$

$$v_t = 0.194 \text{ m/s} < 102 \text{ m/s}$$

The



$$m \frac{dv}{dt} = -mg - \alpha v^2 \quad (1)$$



$$m \frac{dv}{dt} = mg - \alpha v^2 \quad (2)$$

Solve (1) $\Rightarrow \frac{dv}{dt} = -g - \frac{\alpha}{m} v^2 = -g - \beta v^2 \quad (\beta = \frac{\alpha}{m})$

~~$$\int_{t_0}^0 \frac{dv}{(g + \beta v^2)} = \int_0^{t_f} dt$$~~

~~$$v \frac{dv}{dx} = -g - \beta v^2$$~~

$$\therefore \int_{v_0}^0 \frac{v dv}{g + \beta v^2} = \int_0^h -dy$$

~~$$\int_{v_0}^0 \frac{v dv}{1 + \frac{\beta}{g} v^2} = -h$$~~

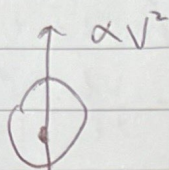
$$\therefore \frac{1}{2\beta} \ln(g + \beta v^2) \Big|_{v_0}^0 = -h \quad \Rightarrow \quad \frac{1}{2\beta} \ln(g + \beta v_0^2) = -h$$

$$\therefore h = \frac{1}{2\beta} \ln \left(\frac{g + \beta v_0^2}{g} \right)$$

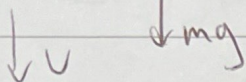
$$= \frac{1}{2\beta} \ln \left(1 + \frac{\beta}{g} v_0^2 \right)$$

$$= \frac{m}{2\alpha} \ln \left(1 + \frac{2v_0^2}{mg} \right) = \frac{a}{2\alpha} \ln \left(1 + \left(\frac{v_0}{v_t} \right)^2 \right)$$

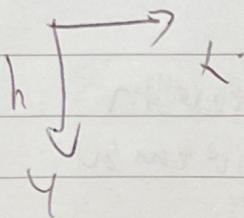
b)



$$m \frac{dv}{dt} = mg - \alpha v^2$$



$$\frac{dv}{dt} = g - \beta v^2$$



$$v \frac{dv}{dy} = g - \beta v^2$$

$$\int_0^{v_r} \frac{v dv}{g - \beta v^2} = \int_0^h dy$$

$$-\frac{1}{2\beta} \ln(g - \beta v_r^2) + \frac{1}{2\beta} \ln(g) = h$$

$$\ln\left(\frac{g}{g - \beta v_r^2}\right) = 2\beta h$$

$$\ln\left(\frac{g}{g - \beta v_r^2}\right) = 2\beta h \quad \therefore \ln\left(\frac{g - \beta v_r^2}{g}\right) = -2\beta h$$

$$\therefore \frac{g - \beta v_r^2}{g} = e^{-2\beta h} = e^{-\frac{2\alpha}{m} h} = e^{-h/a}$$

$$\therefore g e^{-h/a} = e^{-h/a} \beta v_r^2 = g$$

$$\therefore v_r^2 =$$

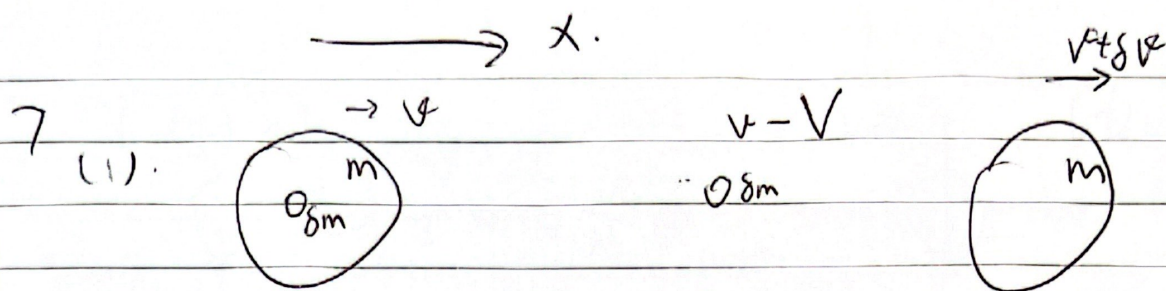
$$\frac{g - \beta v_r^2}{g} = e^{-h/a}$$

$$\therefore g - \beta v_r^2 = g e^{-h/a}$$

$$\therefore v_r^2 = \frac{g}{\beta} (1 - e^{-h/a})$$

$$= \frac{mg}{\alpha} [1 - \exp(-h/a)]$$

$$= v_t^2 [1 - \exp(-h/a)]$$



Conservation of momentum gives

$$(m + \delta m)v = \delta m(v - V) + (v + \delta v)m$$

$$m + \delta m \cancel{v} = \cancel{\delta m}v - \delta mV + \cancel{m}v + m\delta v$$

$$\therefore m\delta v = V\delta m$$

(2) if the small mass is projected forwards:

$$(m + \delta m)v = \delta m(v + V) + (v + \delta v)m$$

$$\therefore m\delta v = -V\delta m$$

$\therefore$ Same expression does not apply.

(3) $\frac{\delta m(t)}{\delta t} = -\frac{\delta m}{\delta t}$ (the mass ~~of~~ ejected ~~plus~~ the mass remaining equals constant)

$$\therefore m \frac{\delta v}{\delta t} = V \left(-\frac{\delta m(t)}{\delta t} \right)$$

$$t \rightarrow 0 \Rightarrow m \frac{dv}{dt} = -V \frac{dm}{dt}$$

$$\therefore \frac{dv}{dt} = -\frac{V}{m} \frac{dm}{dt}$$

The minus appears in the equation indicates that $\frac{dm}{dt}$ is the rate of change of the remaining mass of the rocket, ~~not~~ where as δm is the mass being ejected. The two rates have to be opposite to each other because the total mass of rocket (ejected + remaining) is constant.

(4) with gravity:

$$\frac{dp}{dt} = \cancel{v} - mg \quad \frac{dp}{dt} = -mg$$

$$m \delta v - V \delta m = -mg \delta t.$$

$$\therefore m \frac{\delta v}{\delta t} - V \frac{\delta m}{\delta t} = -mg$$

$$\therefore m \frac{dv}{dt} + V \frac{dm}{dt} = -mg$$

8 (a) According to the last part of Question 7;

$$m \frac{dv}{dt} + V \frac{dm}{dt} = -mg$$

$$\therefore \frac{dm}{dt} = -\alpha.$$

$$\therefore m \frac{dv}{dt} + -\alpha V = -mg$$

$$\therefore \frac{dv}{dt} = \frac{\alpha V}{m} - g$$

$$\therefore m = M - \alpha t.$$

$$\therefore \frac{dv}{dt} = -g + \frac{\alpha V}{M - \alpha t}.$$

$$\begin{aligned} \text{(b)} \quad \frac{dv}{dt} &= -g + \frac{\alpha V}{M - \alpha t} = \frac{-g(M - \alpha t) + \alpha V}{M - \alpha t} \\ &= \frac{\alpha V - Mg - \alpha g t}{M - \alpha t} \end{aligned}$$

$$\text{when } t=0 \quad \left. \frac{dv}{dt} \right|_{t=0} = \frac{\alpha V - Mg}{M}$$

The rocket leaves the launch-pad $\Rightarrow \left. \frac{dv}{dt} \right|_{t=0} > 0$.
 $\Rightarrow \alpha V > Mg$

$$\text{(c)} \quad m_1 = M - \alpha t.$$

$\therefore$ half of M is fuel.

$\therefore$ At time of burn-out. $m = \frac{M}{2}$

$$\frac{M}{2} = M - \alpha t \Rightarrow \alpha t = \frac{M}{2}$$

$$\therefore t = \frac{M}{2\alpha}$$

$$\frac{dv}{dt} = -g + \frac{\alpha V}{m - \alpha t}$$

$$\therefore dv = -g + \frac{\alpha V}{m - \alpha t} dt$$

$$\int_0^V dv = \int_0^{M/2\alpha} -g + \frac{\alpha V}{m - \alpha t} dt$$

$$\therefore V = -gt \Big|_0^{M/2\alpha} - V \ln(m - \alpha t) \Big|_0^{M/2\alpha}$$

$$= -\frac{gM}{2\alpha} - V \ln\left(m - \frac{M}{2}\right) + V \ln(m)$$

$$= V \ln\left(\frac{M}{m/2}\right) - \frac{Mg}{2\alpha}$$

$$= V \ln 2 - \frac{gM}{2\alpha}$$

(c). ~~$\frac{V dv}{dy} = -g + \frac{\alpha V}{m - \alpha t}$~~

$$V = \int_0^t V(t') = \int_0^t -g + \frac{\alpha V}{m - \alpha t'} dt'$$

$$= -gt - V \ln(m - \alpha t) + V \ln(m)$$

$$= V \ln\left(\frac{m}{m - \alpha t}\right) - gt$$

$$\int_0^y dy = \int_0^{M/2\alpha} V \ln\left(\frac{m}{m - \alpha t}\right) - gt dt$$

$$\left\{ \frac{M}{2\alpha} \left(\frac{V m}{m - \alpha t} \ln\left(\frac{m}{m - \alpha t}\right) - \frac{mV}{m - \alpha t} \right) \Big|_0^{M/2\alpha} - \frac{1}{2}gt^2 \Big|_0^{M/2\alpha} \right\}$$

$$\int 2xy \sqrt{1+\frac{y^2}{x^2}} dx.$$

8 (e)

X ↑

$$V(t) = V \ln\left(\frac{M}{M-\alpha t}\right) - gt.$$

$$X = \therefore \cancel{X(t)} = \int_0^X dx = \int_0^t V \ln\left(\frac{M}{M-\alpha t}\right) - gt \, dt$$

$$= -\int_0^t gt \, dt - V \int_0^t \ln\left(1 - \frac{\alpha}{M}t\right) dt.$$

$$= -\left[\frac{1}{2}gt^2\right]_0^t - V \int_0^t \ln\left(1 - \frac{\alpha}{M}t\right) dt$$

$$= -\frac{1}{2}gt^2 - V \int_0^t \ln\left(1 - \frac{\alpha}{M}t\right) dt$$

$$= -\frac{1}{2}gt^2 - V \int_0^t \ln\left(1 - \frac{\alpha}{M}t\right) \cdot 1 \cdot dt$$

$$= -\frac{1}{2}gt^2 - V \left[\left(t \ln\left(1 - \frac{\alpha}{M}t\right) \right) - \int_0^t \frac{-\frac{\alpha}{M}t}{1 - \frac{\alpha}{M}t} dt \right]$$

$$= -\frac{1}{2}gt^2 - V \left[t \ln\left(1 - \frac{\alpha}{M}t\right) \right] + V \int_0^t \frac{-\frac{\alpha}{M}t}{1 - \frac{\alpha}{M}t} dt$$

$$= -\frac{1}{2}gt^2 - Vt \ln\left(1 - \frac{\alpha}{M}t\right) + V \int_0^t \frac{1}{1 - \frac{\alpha}{M}t} dt$$

$$= -\frac{1}{2}gt^2 - Vt \ln\left(1 - \frac{\alpha}{M}t\right) + V \left[t + \frac{M}{\alpha} \ln\left(1 - \frac{\alpha}{M}t\right) \right]_0^t$$

$$= -\frac{1}{2}gt^2 - Vt \ln\left(1 - \frac{\alpha}{M}t\right) + Vt + \frac{MV}{\alpha} \ln\left(1 - \frac{\alpha}{M}t\right)$$

$$= -\frac{1}{2}\left(\frac{M}{2\alpha}\right)^2 - \frac{MV}{2\alpha} \ln\left(\frac{1}{2}\right) + \frac{MV}{2\alpha} + \frac{MV}{\alpha} \ln\left(\frac{1}{2}\right)$$

$$= -\frac{1}{2}\left(\frac{M}{2\alpha}\right)^2 + \frac{MV}{2\alpha} \ln\left(\frac{1}{2}\right) + \frac{MV}{2\alpha}$$

$$= \frac{MV}{2\alpha} \ln 2 - \frac{MV}{2\alpha} \ln 2 - \frac{M^2g}{8\alpha^2}$$

$$= \boxed{\frac{MV}{2\alpha} (1 - \ln 2) - \frac{9M^2}{8\alpha^2}}$$

$$u = \ln\left(1 - \frac{\alpha}{M}t\right)$$

$$du = \frac{1}{1 - \frac{\alpha}{M}t} \left(-\frac{\alpha}{M}\right) dt$$

$$dV = dt$$

$$V = t$$

$$\begin{aligned}
 h &= x + \frac{v^2}{2g} = \frac{Mv}{2\alpha} (1 - \ln 2) - \frac{gm^2}{8\alpha^2} + \frac{(V \ln 2 - gm/2\alpha)^2}{2g} \\
 &= \frac{Mv}{2\alpha} (1 - \ln 2) - \frac{gm^2}{8\alpha^2} + \frac{(V \ln 2)^2}{2g} - \frac{M \cancel{V} g \ln 2}{2g\alpha} + \frac{gm}{8\alpha} \\
 &= \frac{Mv}{2\alpha} (1 - \ln 2) + \frac{(V \ln 2)^2}{2g} - \frac{Mv}{2\alpha} \ln 2 \\
 &= \boxed{\frac{Mv}{2\alpha} (1 - 2 \ln 2) + \frac{(V \ln 2)^2}{2g}}
 \end{aligned}$$

9 (i)

$$M_i = NM$$

$$\begin{aligned} M_f &= NM - (1-r)(N-1)M \\ &= NM - (N - rN + r - 1)M \\ &= (rN - N + rN - r + 1)M \\ &= (rN + (1-r))M \end{aligned}$$

$$\frac{dv}{dt} = -\frac{u}{m} \frac{dm}{dt}$$

$$\therefore \int_0^{V_f} dv = \int_{M_i}^{M_f} -\frac{u}{m} dm \quad V_f = -u \ln \left(\frac{M_f}{M_i} \right) = u \ln \left(\frac{M_i}{M_f} \right)$$

$$V_f = \frac{u}{rN + (1-r)}$$

$$\therefore V_f = u \ln \frac{NM}{(rN + (1-r))M} \Rightarrow \therefore V_f = u \ln \left[\frac{N}{rN + (1-r)} \right]$$

$$\therefore V_f = u \ln \frac{N}{rN + (1-r)}$$

~~Stage 1~~ : $V_1 = u \ln \left[\frac{N}{rN + (1-r)} \right]$

~~Stage 2~~ : $m_i = nm$

$$\begin{aligned} m_f &= nm - (1-r)(n-1)m \\ &= (rn + (1-r))m \end{aligned}$$

$$\frac{dv}{dt} = -\frac{u}{m} \frac{dm}{dt} \Rightarrow \int_{V_1}^{V_2} dv = \int_{m_i}^{m_f} -\frac{u}{m} dm$$

$$V_2 - V_1 = u \ln \left[\frac{n}{rn + (1-r)} \right]$$

$$\therefore V_2 = V_1 + u \ln \left[\frac{n}{rn + (1-r)} \right]$$

$$\therefore V_2 = u \ln \left[\frac{N}{rN + (1-r)} \right] + u \ln \left[\frac{n}{rn + (1-r)} \right]$$

$x \uparrow$ (ii) Stage 1.

$$M_i = NM$$

$$M_f = NM - (1-r)(N-n)M = [rN + n(1-r)]M$$

$$\frac{dv}{dt} = -\frac{u}{m} \frac{dm}{dt} \quad \int_0^{v_1'} dv = -u \ln m \Big|_{M_i}^{M_f}$$

$$v_1' = u \ln \left[\frac{N}{rN + n(1-r)} \right]$$

Stage 2: $M_i' = nm$

$$M_f' = nm - (1-r)(n-1)M = [rn + (1-r)]M$$

$$\therefore \frac{dv}{dt} = -\frac{u}{m} \frac{dm}{dt}$$

$$\int_{v_1}^{v_2} dv = -u \int_{m_i}^{m_f} \frac{1}{m} dm = -u \ln \left(\frac{M_f'}{m_i'} \right)$$

$$\therefore v_2 - v_1 = u \ln \left(\frac{M_i'}{M_f'} \right)$$

$$\therefore v_2 = v_1 + u \ln \left(\frac{M_i'}{M_f'} \right)$$

$$\therefore v_2 = u \ln \left[\frac{N}{rN + n(1-r)} \right] + u \ln \left[\frac{n}{rn + (1-r)} \right]$$

(iii)

$$v_2 = u \ln \left[\frac{Nn}{(rN + n(1-r))(rn + (1-r))} \right]$$

$$= u \ln \left[\frac{Nn}{r^2Nn + rn^2(1-r) + rN(1-r) + n(1-r)^2} \right]$$

$$= -u \ln \left[r^2 + \frac{rn}{N}(1-r) + \frac{r(1-r)}{n} + \frac{(1-r)^2}{N} \right]$$

∴ when V_2 reaches maximum value

$$r^2 + r(1-r)\frac{n}{N} + r(1-r)\frac{1}{n} + \frac{(1-r)^2}{N} \text{ has minimum value}$$

$f(n, N)$

$$f(n, N) = r^2 + (r-r^2)\left(\frac{n}{N} + \frac{1}{n}\right) + (1-r)^2\left(\frac{1}{N}\right)$$

$$\frac{\partial f}{\partial n} =$$

$$V_2 = V_2(n, N)$$

$$\frac{\partial V_2}{\partial n} = -u \left[\frac{rN + n(1-r)}{N} \right] \left(\frac{1}{n} - \frac{r}{N} \right)$$

$$V_2 = -u \ln \left[\frac{rN + n - nr}{N} \right] - u \ln \left[\frac{rN + 1 - r}{n} \right]$$

$$\text{for } \frac{\partial V_2}{\partial n} = -u \left(\frac{N}{rN + n - nr} \right) \left(\frac{1}{n} - \frac{r}{N} \right) = 0$$

$$\frac{u(1-r)}{rN + n - nr} = \frac{n}{rN + 1 - r} = 0$$

$$V_2 = -u \ln \left[r + \frac{n}{N} - \frac{r}{N}n \right] - u \ln \left[r + \frac{1}{n} - \frac{r}{N} \right]$$

$$\frac{\partial V_2}{\partial n} = -u \left(\frac{1}{r + \frac{n}{N} - \frac{r}{N}n} \right) \left(\frac{1}{N} - \frac{r}{N} \right)$$

$$-u \left(r + \frac{1}{n} - \frac{r}{N} \right) \left(-\frac{1}{n^2} + \frac{r}{N^2} \right) = 0$$

$$\therefore \frac{\frac{1}{N} - \frac{r}{N}}{r + \frac{n}{N} - \frac{r}{N}n} + \frac{-\frac{1}{n^2} + \frac{r}{N^2}}{r + \frac{1}{n} - \frac{r}{N}} = 0$$

$$\therefore \frac{1-r}{rN + n - nr} + \frac{-(1-r)}{n^2 + n - nr} = 0$$

$$\therefore \text{when } \boxed{N = n^2} \quad V_2 \text{ has maximum}$$

$$V_2^{\max} = u \ln \left[\frac{n^2}{rn^2 + k(1-r)} \right] + u \ln \left[\frac{n}{rn + (1-r)} \right]$$

$$= 2u \ln \left[\frac{n}{rn + (1-r)} \right]$$

(iv) The earth's escape speed $V_e = 7.9 \text{ km s}^{-1}$

2 stage ~~rocket~~ : let $V_2^{\max} = V_e$
 rocket $V_e = 2u \ln \left[\frac{n}{rn + (1-r)} \right]$

$$\therefore \frac{n}{rn + (1-r)} = e^{V_e/2u}$$

$$\therefore n = (e^{V_e/2u}) (rn + e^{V_e/2u} - e^{V_e/2u} r)$$

~~$$\therefore (re^{V_e/2u} - 1)n =$$~~

$$\therefore (1 - re^{V_e/2u})n = (1-r)e^{V_e/2u}$$

$$\therefore n = \frac{e^{V_e/2u} (1-r)}{1 - re^{V_e/2u}} \approx 5.76$$

$$N = 5.76^2 = 33.18$$

Q | stage rocket :

$$N = \frac{e^{V_e/2u} (1-r)}{1 - re^{V_e/2u}} = -0.52 < 0$$

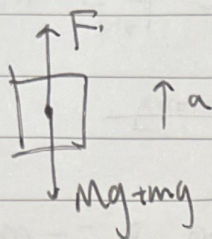
$\therefore$ for ~~stage~~ one stage rockets the earth's escape speed cannot be achieved, for two stage rockets it can be ~~achieved~~ achieved as long as $N \geq 33.18$

use:
11.2

10.

(a) $F = ma \therefore a =$

$\Sigma F = M \cdot (m+m) a.$



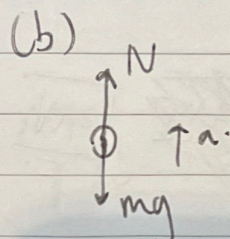
$\therefore F - Mg = M a$

$\therefore F - (M+m)g = (M+m)a.$

$\therefore a = \frac{F - (M+m)g}{M+m}$

$\therefore M \gg m$

$\therefore a \approx \frac{F - Mg}{M} = \frac{F}{M} - g$



$N - mg = ma.$

$\therefore N = mg + ma.$

$= mg + m \left(\frac{F}{M} - g \right)$

$= mg + \frac{m}{M} F - mg$

$= \frac{m}{M} F$

$\therefore$ apparent weight of parcel is $\frac{m}{M} F$

(c) Lab Frame :

$y_1 = V_0 t - \frac{1}{2} g t^2$

$y_1 = V_0 t + \frac{1}{2} a t^2$

$y_2 = V_0 t - \frac{1}{2} g t^2$

$y_1 = V_0 t + \frac{1}{2} a t^2$

$\therefore y_1 + y_2 = \frac{1}{2} (a+g) t^2$

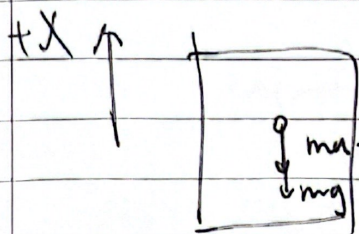
$y_1 + y_2 = h$

$\therefore h = \frac{1}{2} (a+g) t^2$

$\therefore t = \sqrt{\frac{2h}{a+g}} = \sqrt{\frac{2h}{\frac{F}{M} - g}}$

$\therefore t = \sqrt{\frac{2Mh}{F}}$

Lift Frame:



fictitious force $F = -ma$.

~~F_{fict}~~

~~$\ddot{x} = -a - g$~~

$$\Sigma F = m\ddot{x} = F + G$$

$$= -ma - mg = -m(a+g)$$

$$\therefore \ddot{x} = \Sigma F/m = -(a+g)$$

$$\Delta x = \frac{1}{2} \ddot{x} t^2$$

$$\therefore -h = \frac{1}{2} (-(a+g)) t^2$$

$$\therefore h = \frac{1}{2} (a+g) t^2$$

$$\therefore t = \sqrt{\frac{2h}{a+g}} = \sqrt{\frac{2Mh}{F}}$$

(d) Lift changes to constant velocity $\Rightarrow a=0$.

$$\therefore 0 = \frac{F}{m} - g \quad \therefore F = Mg$$

11 (a) free fall from infinity:

$$0 + 0 = \frac{1}{2} m v_1^2 - \frac{GMm}{R_E}$$

$$\therefore v_1 = \sqrt{\frac{2GM_E}{R_E}}$$

constant acceleration.

$$v_2^2 - 0^2 = 2gR_E \quad \therefore v_2 = \sqrt{2gR_E}$$

$$\therefore v_2 = \sqrt{\frac{2GM_E R_E}{R_E^2}} = \sqrt{\frac{2GM_E}{R_E}}$$

$$\therefore v_1 = v_2$$

(b)

$$g - \frac{GMEm}{(R_E + h)} = \frac{1}{2}mv^2 - \frac{GMEm}{R_E}$$

$$\therefore \frac{1}{2}mv^2 = \frac{GMEm}{R_E} - \frac{GMEm}{R_E} \left(1 + \frac{h}{R_E}\right)^{-1}$$

$$= \frac{GMEm}{R_E} \left(1 - \left(1 - \frac{h}{R_E} + \frac{h^2}{R_E^2}\right)\right)$$

$$= 0 \quad mgR_E \left(\frac{h}{R_E} - \frac{h^2}{R_E^2}\right)$$

$$= mgh - \frac{mgh^2}{R_E} = mgh \left(1 - \frac{h}{R_E}\right)$$

$$\therefore v^2 = 2gh \left(1 - \frac{h}{R_E}\right)$$

$$\therefore v = \sqrt{2gh} \left(1 - \frac{h}{R_E}\right)^{1/2}$$

$$= \sqrt{2gh} \left(1 - \frac{h}{2R_E}\right)$$

(c)

$$\cancel{T = 2\pi\sqrt{\frac{l}{g}}} \quad T = 2\pi\sqrt{\frac{l}{g}} \quad T' = 2\pi\sqrt{\frac{l}{g'}}$$

$$\therefore \Delta T = T' - T = 2\pi\sqrt{l} \left(g'^{-1/2} - g^{-1/2}\right)$$

$$T = (2\pi\sqrt{l})(g^{-1/2})$$

$$\therefore \frac{\Delta T}{T} = \frac{g'^{-1/2} - g^{-1/2}}{g^{-1/2}} = \sqrt{\frac{g}{g'}} - 1$$

$$g = \frac{GM_E}{R_E^2} \quad g' = \frac{GM_E}{(R_E + h)^2} = \frac{GM_E}{R_E^2} \left(1 + \frac{h}{R_E}\right)^{-2} = \frac{GM_E}{R_E^2} \left(1 - \frac{2h}{R_E}\right)$$

$$\therefore \frac{g'}{g} = \frac{\frac{GM_E}{R_E^2} \left(1 - \frac{2h}{R_E}\right)}{\frac{GM_E}{R_E^2}} = 1 - \frac{2h}{R_E}$$

$$\therefore \sqrt{\frac{g}{g'}} = \left(\frac{g}{g'}\right)^{1/2} = \left(\frac{g'}{g}\right)^{-1/2} = \left(1 - \frac{2h}{R_E}\right)^{-1/2} = 1 + \frac{h}{R_E}$$

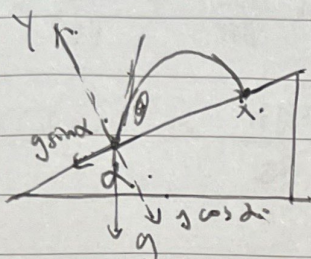
$$\therefore \frac{\Delta T}{T} = \sqrt{\frac{g}{g'}} - 1 = \left(1 + \frac{h}{R_E}\right) - 1 = \frac{h}{R_E} \quad \therefore \frac{\Delta T}{T} = \frac{h}{R_E}$$

$\therefore \Delta T > 0 \quad T' > T$ $\therefore$ clock-gains time.

$$\Delta T = \frac{h}{P_{Fe}} T = \frac{100 \text{ m}}{6370 \times 10^3 \text{ m}} (7 \times 24 \times 60 \times 60) / \text{week}.$$

$$= 9.55 / \text{week}$$

12 a.



$$X = V \cos \theta t - \frac{1}{2} g \sin \alpha t^2$$

$$Y = V \sin \theta t - \frac{1}{2} g \cos \alpha t^2$$

for $Y = 0 \quad (t \neq 0)$

$$V \sin \theta t = \frac{1}{2} g \cos \alpha t^2$$

$$\therefore t = \frac{2V \sin \theta}{g \cos \alpha} \quad \because \theta = \alpha = 30^\circ \quad \therefore t = \frac{2V}{g}$$

$$X = V \cos \theta t - \frac{1}{2} g \sin \theta t^2$$

$$= V \left(\frac{\sqrt{3}}{2} \right) \left(\frac{2}{g} \right) - \frac{1}{2} g \left(\frac{1}{2} \right) \left(\frac{2V}{g} \right)^2$$

$$= \frac{V^2}{g}$$

$$X = V \cos \theta t - \frac{1}{2} g \sin \alpha t^2$$

$$= \frac{\sqrt{3}}{2} V \left(\frac{2V}{g} \right) - \frac{1}{2} g \left(\frac{1}{2} \right) \left(\frac{2V}{g} \right)^2$$

$$= \frac{V^2}{g} - \frac{1}{4} g \frac{4V^2}{3g} = \frac{V^2}{g} - \frac{V^2}{3g} = \frac{2V^2}{3g}$$

b. upwards:

$$R = V \sin \theta t - \frac{1}{2} g \sin \theta t^2$$

$$R = V \cos \theta t - \frac{1}{2} g \sin \theta t^2$$

$$= V \cos \theta \left(\frac{2V \sin \theta}{g \cos \alpha} \right) - \frac{1}{2} g \sin \theta \left(\frac{2V \sin \theta}{g \cos \alpha} \right)^2$$

$$= \frac{2V^2 \sin \theta \cos \theta}{\cos \alpha} - \frac{1}{2} g \sin \theta \frac{4V^2 \sin^2 \theta}{g^2 \cos^2 \alpha}$$

$$= \frac{W^2 \sin \theta \cos \theta}{g \cos \alpha} - \frac{W^2 g \sin^2 \theta \sin \alpha}{g^2 \cos^2 \alpha}.$$

$$= \frac{W^2 \sin \theta \cos \theta}{g \cos \alpha} - \frac{W^2 \sin^2 \theta \sin \alpha}{g \cos^2 \alpha}.$$

$$= \frac{W^2}{g} \left(\frac{\sin \theta \cos \theta \cos \alpha - \sin^2 \theta \sin \alpha}{\cos^2 \alpha} \right).$$

$$= \frac{W^2}{g} \left(\frac{\sin \theta (\cos \theta \cos \alpha - \sin \theta \sin \alpha)}{\cos^2 \alpha} \right).$$

$$= \frac{W^2}{g} \left(\frac{\sin \theta \cos (\theta + \alpha)}{\cos^2 \alpha} \right).$$

$$= \frac{W^2}{g \cos^2 \alpha} \sin \theta \cos (\theta + \alpha)$$

$$R'(\theta) = \frac{W^2}{g \cos^2 \alpha} [-\sin \theta \sin (\theta + \alpha) + \cos \theta \cos (\theta + \alpha)].$$

$$= \frac{W^2}{g \cos^2 \alpha} [\cos (2\theta + \alpha)].$$

$$\therefore \text{for } R'(\theta) = 0$$

$$\text{for } R_{up} = R_{max}^{up}, \quad R'(\theta) = 0.$$

$$\therefore \cos (2\theta + \alpha) = 0 \quad \therefore 2\theta + \alpha = \frac{\pi}{2}.$$

$$\therefore \theta = \frac{\pi}{4} - \frac{\alpha}{2}.$$

$$\therefore R_{up} = \frac{W^2}{g \cos^2 \alpha} \sin \left(\frac{\pi}{4} - \frac{\alpha}{2} \right) \cos \left(\frac{\pi}{4} + \frac{\alpha}{2} \right)$$

$$= \frac{W^2}{g \cos^2 \alpha} \left[\sin \left(\frac{\pi}{2} - \alpha \right) \cos \left(\frac{\pi/2 + \alpha}{2} \right) \right]$$

$$= \frac{W^2}{g \cos^2 \alpha} (\sin \frac{\pi}{2} - \sin \alpha)$$

$$= \frac{W^2}{g \cos^2 \alpha} (1 - \sin \alpha) = \frac{V^2}{g \cos^2 \alpha} (1 - \sin \alpha).$$

downwards :
$$\begin{cases} X = V \cos \theta t + \frac{1}{2} g \sin \alpha t^2 \\ Y = V \sin \theta t - \frac{1}{2} g \cos \alpha t^2 \end{cases}$$

$$\therefore t = \frac{2V \sin \theta}{g \cos \alpha}$$

$$\therefore R = \frac{2V^2}{g \cos^2 \alpha} (\sin \theta \cos(\theta - \alpha))$$

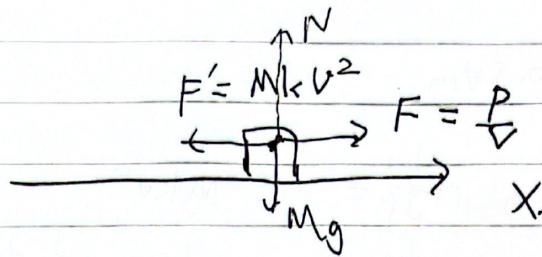
$$\begin{aligned} R'(\theta) &= \frac{2V^2}{g \cos^2 \alpha} (\cos \theta \cos(\theta - \alpha) + \sin \theta (-\sin(\theta - \alpha))) \\ &= \frac{2V^2}{g \cos^2 \alpha} (\cos(2\theta - \alpha)) \end{aligned}$$

$2\theta - \alpha = \frac{\pi}{2}$ $\therefore \theta = \frac{\pi}{4} + \frac{\alpha}{2}$ $\rightarrow (R_{\text{down}} \rightarrow \text{max.})$

$$\begin{aligned} \therefore R_{\text{down}} &= \frac{2V^2}{g \cos^2 \alpha} \left(\sin \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \cos \left(\frac{\pi}{4} - \frac{\alpha}{2} \right) \right) \\ &= \frac{2V^2}{g \cos^2 \alpha} \left(2 \left(\sin \left(\frac{\pi/2 + \alpha}{2} \right) \cos \left(\frac{\pi/2 - \alpha}{2} \right) \right) \right) \\ &= \frac{V^2}{g \cos^2 \alpha} (\sin \frac{\pi}{2} + \sin \alpha) \\ &= \frac{V^2}{g \cos^2 \alpha} (1 + \sin \alpha) \end{aligned}$$

$$\begin{aligned} \therefore \frac{R_{\text{up}}}{R_{\text{down}}} &= \frac{(V^2 / g \cos^2 \alpha) (1 - \sin \alpha)}{(V^2 / g \cos^2 \alpha) (1 + \sin \alpha)} \\ &= \frac{1 - \sin \alpha}{1 + \sin \alpha} \end{aligned}$$

13 (a).



At terminal speed the body travels at a constant speed $\Rightarrow$ forces acting on the body are equal.

$$\therefore F + F' = 0 \therefore \frac{P}{V_m} - MKV_m^2 = 0$$

$$\therefore \boxed{V_m^3 = \frac{P}{MK}}$$

$$(b) \quad m \frac{dv}{dt} = \frac{P}{V} - MKV^2$$

$$\therefore mV \frac{dv}{dx} = \frac{P}{V} - MKV^2$$

$$\therefore mV^2 \frac{dv}{dx} = P - MKV^3$$

$$\therefore V^2 dv = \left(\frac{P}{m} - KV^3 \right) dx$$

$$\therefore \int_0^{V'} \frac{V^2 dv}{\frac{P}{m} - KV^3} = \int_0^X dx$$

$$\therefore -\frac{1}{3K} \ln \left(\frac{P}{m} - KV^3 \right) \Big|_0^{V'} = X$$

$$\therefore X = -\frac{1}{3K} \left(\frac{P}{m} - KV'^3 \right) + \frac{1}{3K} \left(\frac{P}{m} \right)$$

$$X = \frac{1}{3K} \ln \left(\frac{P/m}{P/m - KV'^3} \right)$$

$$V' = \frac{V_m}{2} \therefore V'^3 = \frac{V_m^3}{8} = \frac{P}{8MK}$$

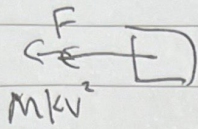
$$\therefore X = \frac{1}{3K} \ln \left(\frac{P/m}{P/m - K(P/8MK)} \right) = \frac{1}{3K} \ln \left(\frac{P/m}{P/m - P/8M} \right)$$

$$= \frac{1}{3K} \ln \left(\frac{P/m}{\frac{7}{8}P/m} \right) = \boxed{\frac{1}{3K} \ln(8/7)}$$

$\frac{1}{3K} \ln \left(\frac{P/m}{P/m - KV^3} \right)$
 $\frac{1}{3K} \ln \left(\frac{P/m}{P/m - KV^3} \right)$
 $\frac{1}{3K} \ln \left(\frac{P/m}{P/m - KV^3} \right)$

$$\frac{\sqrt{\frac{mk}{F}}}{1 + \left(\sqrt{\frac{mk}{F}} v\right)^2} \leftarrow \tan^{-1} \left(\sqrt{\frac{mk}{F}} v\right)$$

(C) $V_i = 0.5 \text{ Vm}$



$$\therefore m \frac{dv}{dt} = -F - mkv^2$$

$$\frac{dv}{F + mkv^2} = -\frac{1}{m} dt$$

$$\int_{V_i}^0 \frac{dv}{1 + \frac{mk}{F} v^2} = -\int_0^t \frac{F}{m} dt$$

$$\therefore \sqrt{\frac{mk}{F}} \left[\tan^{-1} \left(\sqrt{\frac{mk}{F}} v \right) \right]_{V_i}^0 = -\int_0^t \frac{F}{m} dt$$

$$\therefore \sqrt{\frac{F}{mk}} \tan^{-1} \left(\sqrt{\frac{mk}{F}} V_i \right) = \frac{F}{m} t \quad \therefore V_i = 0.5 \text{ Vm}$$

$$\therefore t = \sqrt{\frac{m}{Fk}} \tan^{-1} \left(\left(\sqrt{\frac{mk}{F}} \right) 0.5 \text{ Vm} \right)$$

$$1 + \sigma v^2 \sqrt{v^3}$$

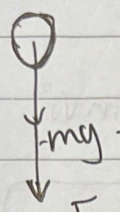
$$\frac{v^3 + \frac{1}{\sigma} v}{6v^2 + 1}$$

$$\frac{x^3}{1+x^2} \cdot \frac{x^3+x-x}{1+x^2}$$

14

(a). W_{nc} = work done by non conservative force.

$$W_{nc} = \int F \cdot dx = \int_0^h F \cdot dx.$$



$$= \int_0^h -\alpha v^2 dx.$$

$$m \frac{dv}{dt} = -mg - \alpha v^2.$$

$$m v \frac{dv}{dx} = -mg - \alpha v^2.$$

$$\therefore v dv = (-g - \frac{\alpha}{m} v^2) dx.$$

$$\therefore dx = - \frac{v dv}{g + \frac{\alpha}{m} v^2}.$$

$$\therefore F = \int_{v_0}^0 \frac{+\alpha v^2 (v dv)}{g + \frac{\alpha}{m} v^2} = \alpha \int_{v_0}^0 \frac{v^3}{g + \frac{\alpha}{m} v^2} dv.$$

$$= \frac{\alpha}{g} \int_{v_0}^0 \frac{v^3}{1 + \frac{\alpha}{mg} v^2} dv. \quad (\text{let } \frac{\alpha}{mg} = \sigma).$$

$$= \frac{\alpha}{g} \int_{v_0}^0 \frac{v^3}{1 + \sigma v^2} dv = \frac{\alpha}{g\sigma} \int_{v_0}^0 \frac{\sigma v^3}{1 + \sigma v^2} dv$$

$$= \frac{\alpha}{g\sigma} \int_{v_0}^0 \frac{\sigma v^3 + v}{1 + \sigma v^2} dv - \frac{\alpha}{g\sigma} \int_{v_0}^0 \frac{v}{1 + \sigma v^2} dv$$

$$= \frac{\alpha}{g\sigma} \int_{v_0}^0 v dv - \frac{\alpha}{g\sigma} \int_{v_0}^0 \frac{v}{1 + \sigma v^2} dv.$$

$$= \frac{\alpha}{g\sigma} \left(\frac{1}{2} v^2 \right) \Big|_{v_0}^0 - \frac{\alpha}{g\sigma} \left(\frac{1}{2\sigma} \right) \left(\ln(1 + \sigma v^2) \right) \Big|_{v_0}^0$$

$$= -\frac{\alpha}{2g\sigma} v_0^2 - \frac{\alpha}{2g\sigma^2} (0 - \ln(1 + \sigma v_0^2))$$

$$\ln(1 + \sigma v^2) \cdot \frac{2\sigma v}{1 + \sigma v^2} \cdot (1 + \sigma v^2)$$

$$\therefore W_{nc} = \frac{\alpha}{2g\alpha^2} \ln(1 + \alpha V_0^2) - \frac{\alpha}{2g\alpha} V_0^2 \quad \because \alpha = \frac{\alpha}{mg}$$

$$\therefore W_{nc} = \frac{\frac{\alpha m^2 g}{2g\alpha^2}}{\ln(1 + \alpha V_0^2)} - \frac{\alpha mg}{2g\alpha} V_0^2$$

$$= \frac{\frac{1}{2} m^2 g}{\alpha} \ln(1 + \frac{\alpha V_0^2}{mg}) - \frac{1}{2} m V_0^2$$

$$= \boxed{\frac{1}{2} m V_0^2 \ln(1 + \frac{V_0^2}{V_1^2}) - \frac{1}{2} m V_0^2} \quad \begin{array}{l} \text{energy} \\ \text{dissipation} \end{array}$$

$$\Delta T + W_c + W_{nc} = -mgh + W_{nc}$$

$$\cancel{\frac{1}{2} m g h} = -mg \left(a \ln(1 + \frac{V_0^2}{V_1^2}) \right) + W_{nc} \quad (a = \frac{V_1^2}{2g})$$

$$= -mg \frac{V_1^2}{2g} \ln(1 + \frac{V_0^2}{V_1^2}) + \frac{1}{2} m V_0^2 \ln(1 + \frac{V_0^2}{V_1^2}) - \frac{1}{2} m V_0^2$$

$$= -\frac{1}{2} m V_0^2 = \Delta T$$

$$\therefore \Delta T = W_c + W_{nc} = \cancel{\Delta T} + W$$

$\therefore$ Consistent.

(b).

$$\text{for } V_0 = V_1 \Rightarrow h = a \ln(1 + \frac{V_0^2}{V_1^2})$$

$$= a \ln(1 + 1) = a \ln 2 \quad \therefore \frac{h}{a} = \ln 2$$

From Q6 we know that.

$$V_1^2 = V_0^2 [1 - \exp(-h/a)]$$

$$= V_0^2 [1 - \exp(-\ln 2)]$$

$$= V_0^2 (1 - \frac{1}{2}) = \frac{1}{2} V_0^2$$

$\therefore$ ~~the~~ initial ~~kinetic~~ energy T_i , final energy T_f ,

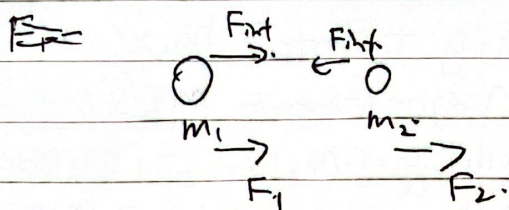
$$\Rightarrow T_i = \frac{1}{2} m v_0^2$$

$$T_f = \frac{1}{2} m v_f^2$$

$$\therefore \frac{T_f}{T_i} = \frac{\frac{1}{2} m v_f^2}{\frac{1}{2} m v_0^2} = \frac{v_f^2}{v_0^2} = \frac{\frac{1}{2} v_0^2}{v_0^2} = \frac{1}{2}$$

$\therefore$ half of the initial energy is lost during the round trip.

15



$$F_1 + F_{int} = m_1 \ddot{x}_1 \quad F_2 - F_{int} = m_2 \ddot{x}_2$$

~~F_2~~

$$\star F_1 + F_2 = m_1 \ddot{x}_1 + m_2 \ddot{x}_2$$

From Q6-I we know that

$$\ddot{x}_c = \frac{m_1 \ddot{x}_1 + m_2 \ddot{x}_2}{m_1 + m_2} \quad \therefore m_1 \ddot{x}_1 + m_2 \ddot{x}_2 = (m_1 + m_2) \ddot{x}_c$$

$$\therefore F_1 + F_2 = (m_1 + m_2) \ddot{x}_c$$

$$\begin{aligned} \therefore F_1 &= m_1 g \\ F_2 &= m_2 g \end{aligned} \quad \therefore m_1 g + m_2 g = (m_1 + m_2) \ddot{x}_c$$

$$\therefore \ddot{x}_c (m_1 + m_2) = g (m_1 + m_2)$$

$$\therefore \ddot{x}_c = g$$

$$m_1 g + F_{int} = m_1 \ddot{x}_1$$

$$m_2 g - F_{int} = m_2 \ddot{x}_2$$

$$\text{In CM Frame: } x_1' = x_1 - x_{cm} \\ = x_1 - \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{(x_1 - x_2) m_2}{m_1 + m_2}$$

$$x_2' = x_2 - x_{cm} = x_2 - \frac{m_1 x_1 + x_2 m_2}{m_1 + m_2}$$

$$= \frac{-m_1 (x_1 - x_2)}{m_1 + m_2}$$

$$F_{int} = m_1 \ddot{x}_1' - \ddot{x}_2' = \frac{m_1 m_2}{m_1 + m_2} (\ddot{x}_1' - \ddot{x}_2')$$

$$m_1 g + F_{int} = m_1 \ddot{x}_1'$$

$$m_2 g - F_{int} = m_2 \ddot{x}_2'$$

$$\therefore m_1 m_2 g + m_2 F_{int} = m_1 m_2 \ddot{x}_1' \quad (1)$$

$$m_1 m_2 g - m_1 F_{int} = m_1 m_2 \ddot{x}_2' \quad (2)$$

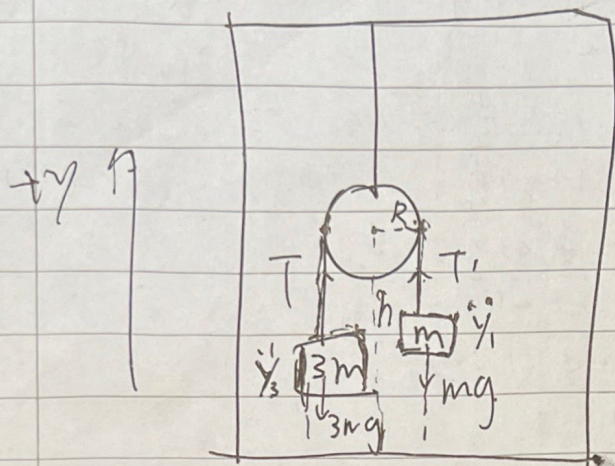
$$(1) - (2) \Rightarrow F_{int} (m_1 + m_2) = m_1 m_2 (\ddot{x}_1' - \ddot{x}_2')$$

$$\therefore F_{int} = \frac{m_1 m_2}{m_1 + m_2} (\ddot{x}_1' - \ddot{x}_2')$$

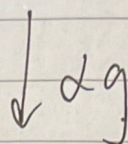
$$a = \ddot{x}_1' - \ddot{x}_2'$$

$$a' = \ddot{x}_1' - \ddot{x}_2' \quad \therefore F_{int} = \mu a = \mu a'$$

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(a)



$$T - mg = m\ddot{y}_1 \quad (1)$$

$$T - 3mg = 3m\ddot{y}_3 \quad (2)$$

$$h - y_1 + h - y_3 + 2R = l \quad (3)$$

$$(3) \Rightarrow 2h = y_1 + y_3 - 2R + l$$

$$2\dot{h} = \dot{y}_1 + \dot{y}_3$$

$$\ddot{h} = -\alpha g \quad \therefore \ddot{y}_1 + \ddot{y}_3 = -2\alpha g$$

$$(b). -3mg + mg = 3m\ddot{y}_3 - m\ddot{y}_1$$

$$\therefore -2mg = m(3\ddot{y}_3 - \ddot{y}_1)$$

$$2\ddot{y}_3 - \ddot{y}_1 = -2g$$

(c)

$$\ddot{y}_1 + \ddot{y}_3 = -2\alpha g$$

(d)

$$\cancel{4\ddot{y}_3 = -2g(1+\alpha)}$$

inertial.

$$4\ddot{y}_3 = -2g(1+\alpha)$$

$$\therefore \ddot{y}_3 = -\frac{1}{2}g(1+\alpha)$$

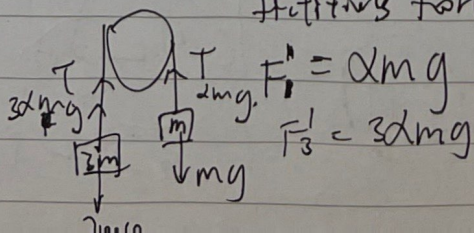
$$\therefore \ddot{y}_1 = -\ddot{y}_3 - 2\alpha g$$

$$= \frac{1}{2}g(1+\alpha) - 2\alpha g = \frac{1}{2}g(1-3\alpha)$$

$$T = mg + \frac{1}{2}mg - \frac{3}{2}mg = \frac{3}{2}mg(1-\alpha)$$

non-inertial.

frictionless force.



$$\therefore T - 3m(1-\alpha)g = 3ma$$

$$T - m(1-\alpha)g = ma$$

$$\therefore 2m(1-\alpha)g = 4ma$$

$$\ddot{y}_3 = -a - \alpha g = -\frac{1}{2}(1-\alpha)g - \alpha g = -\frac{1}{2}(1+\alpha)g$$

$$\ddot{y}_1 = a - \alpha g = \frac{1}{2}(1-\alpha)g - \alpha g = \frac{1}{2}(1-3\alpha)g$$

$$\text{for } \alpha = 1/3 \quad a = \frac{1}{2}\left(\frac{2}{3}\right)g = \frac{g}{3}$$

$$\ddot{y}_3 = -\frac{1}{2}\left(1+\frac{1}{3}\right)g = -\frac{2}{3}g \quad \ddot{y}_1 = \frac{1}{2}(1-3\alpha)g = 0$$

$$\text{for } \alpha = 1 \quad a = \frac{1}{2}(0)g = 0$$

$$\ddot{y}_3 = -\frac{1}{2}(2)g = -g$$

$$\ddot{y}_1 = \frac{1}{2}(1-3\alpha)g = -g$$

$$T = \frac{3}{2}mc(1-\alpha)g$$

(e) Yes.

The equations are the same as those in (d)

