EM5 Ziyan Li

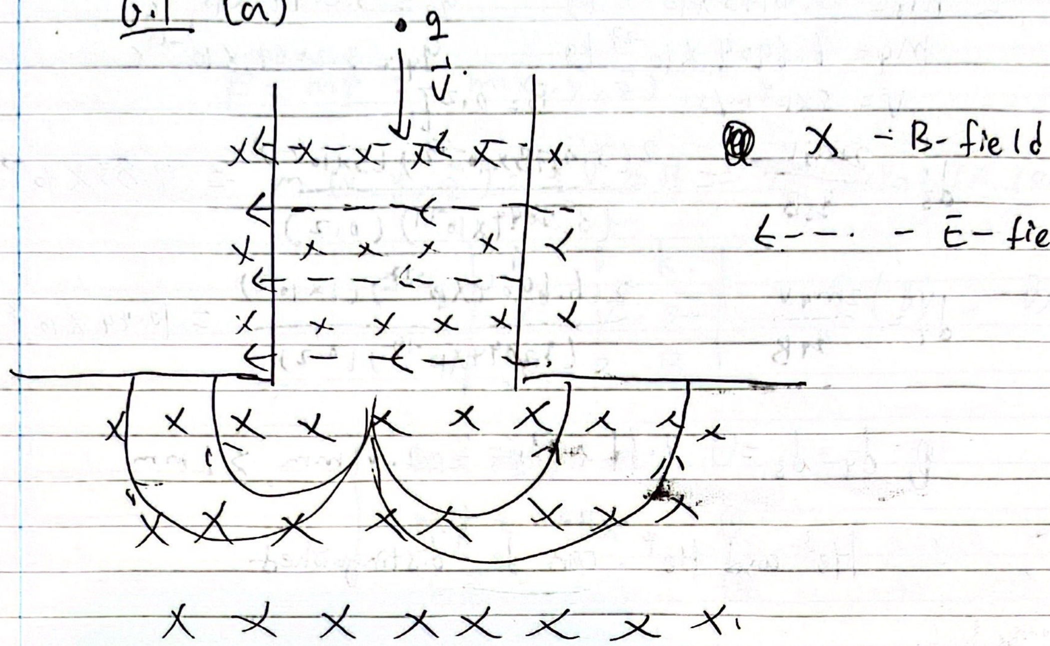
Q.0

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$\vec{F}$ is force q is charge $\vec{v}$ is velocity

$\vec{E}$ is electric field. $\vec{B}$ is magnetic flux density

Q.1 (a)

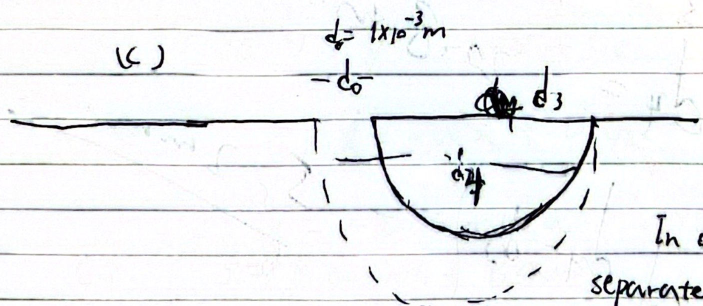


(b) For the particle to pass through the filter the electric force and magnetic force acting on the charged particle must be balanced. According to the set-up in (a),

$\therefore qE = qvB$ $\therefore$ $v = \frac{E}{B} = \frac{100\,V/cm}{0.2T} = \frac{10000\,V/m}{0.2T}$

$= \boxed{5 \times 10^4 \, m/s}$

(c)



mass of $He^3 = m_3$, charge q_3
mass of $He^4 = m_4$, charge q_4

In order for the machine to be able to separate He^3 and He^4. The diameter of trajectories must satisfy $|d_3 - d_4| > d_0 = 1\times10^{-3}\,m$. The radii are $r_3 = d_3/2$, $r_4 = d_4/2$

Motion in B-field (velocity perpendicular to $\vec{B}$ initially):

$F = qvB = \frac{v^2}{r}m$ $\therefore$ $r = \frac{mv}{qB}$ $\therefore$ $d = \frac{2mv}{qB}$

$\therefore$ $\therefore$ $d_3 = \frac{2m_3 v}{q_3 B}$ $d_4 = \frac{2m_4 v}{q_4 B}$

2

He^3 has 2 protons and 1 neutron.
He^4 has 2 protons and 2 neutrons
(so no electrons involved)

Assume ions are nucleus and assume mass of proton equals to mass of neutron $= m_p = 1.6726 \times 10^{-27}$ kg. elementary charge $e = 1.6022 \times 10^{-19}$ C.

Then $m_3 = 3m_p$ $m_4 = 4m_p$ $q_3 = q_4 = 2e$

$\therefore m_3 = 5.0178 \times 10^{-27}$ kg $q_3 = 3.2044 \times 10^{-19}$ C.
$m_4 = 6.6904 \times 10^{-27}$ kg $q_4 = 3.2044 \times 10^{-19}$ C.
$V = 5 \times 10^4$ m/s $B = 0.2T$

$$d_3 = \frac{2m_3V}{q_3B} = \frac{2(5.0178 \times 10^{-27})(5 \times 10^4)}{(3.2044 \times 10^{-19})(0.2)} = 7.83 \times 10^{-3} m.$$

$$d_4 = \frac{2m_4V}{q_4B} = \frac{2(6.6904 \times 10^{-27})(5 \times 10^4)}{(3.2044 \times 10^{-19})(0.2)} = 10.44 \times 10^{-3} m.$$

$d_4 - d_3 = 2.61 \times 10^{-3} m = 2.61 mm > 1 mm$

$\therefore$ He^3 and He^4 <u>can be</u> distinguished.

$q_3 = q_4 = e$

$d_3 = 15.6$ mm
$d_4 = 20.8$ m.m.

$|d_4 - d_3| = 5.2$ mm.

G.2 (a)

In a magnetic field.

$\vec{F} = q\vec{V}\times\vec{B}$ ~~Initial conditions~~ :

$\vec{F} = (F_x, F_y, F_z)$

$\vec{V} = (V_x, V_y, V_z) = (\dot{x}, \dot{y}, \dot{z})$

$\vec{B} = (B_x, B_y, B_z)$

If we orient the z-axis to contain $\vec{B}$, then $B_x = B_y = 0$

we set $B_z = B$

$\therefore \vec{B} = (0, 0, B)$

$\vec{F} = m\ddot{\underline{r}} = m(\ddot{x}, \ddot{y}, \ddot{z})$

$\therefore m(\ddot{x}, \ddot{y}, \ddot{z}) = q\vec{V}\times B = q(\dot{x}, \dot{y}, \dot{z}) \times (0, 0, B)$

$$= q\begin{vmatrix} i & j & k \\ \dot{x} & \dot{y} & \dot{z} \\ 0 & 0 & B \end{vmatrix} = q(B\dot{y}, -B\dot{x}, 0)$$

$$\therefore \begin{cases} m\ddot{x} = qB\dot{y} & ① \\ m\ddot{y} = -qB\dot{x} & ② \\ m\ddot{z} = 0 & ③ \end{cases}$$

Integrate ①, ②, and ③ gives

$\dot{x} = \omega y + C_1 \qquad \dot{y} = -\omega x + C_2$

$\dot{z} = C$ let $\boxed{\omega = \frac{qB}{m}}$

$z = ct + d$

$\because \dot{x} = \omega y + c \quad \therefore \ddot{x} = \omega\dot{y} \quad \therefore \dot{y} = \frac{\ddot{x}}{\omega}$

$\because \dot{y} = -\omega x + C_2 \quad \therefore \frac{\ddot{x}}{\omega} = -\omega x + C_2$

$\therefore \ddot{x} = -\omega^2 x + \omega C_2 = -\omega^2 x + \omega^2 a \quad \boxed{(a = \frac{C_2}{\omega})}$

$\therefore \ddot{x} = -\omega^2 x + \omega^2 a \Rightarrow \ddot{x} + \omega^2 x = \omega^2 a$

C.F $x = A\cos\omega t + B\sin\omega t$

P.I $x = a$

$\therefore \boxed{x = a + A\cos\omega t + B\sin\omega t}$

$\therefore \dot{x} = -\omega A\sin\omega t + \omega B\cos\omega t = \omega y + C_1$

$\therefore \boxed{y = b + B\cos\omega t - A\sin\omega t} \quad (b = -\frac{C_1}{\omega})$

$\boxed{z = ct + d}$

Projection on the x-y plane:

$$(x-a)^2 + (y-b)^2 = (A\cos\omega t + B\sin\omega t)^2 + (B\cos\omega t - A\sin\omega t)^2$$

$$= A^2(\cos^2\omega t + \sin^2\omega t) + B^2(\cos^2\omega t + \sin^2\omega t)$$
$$+ 2AB\cos\omega t\sin\omega t - 2AB\cos\omega t\sin\omega t$$

$$= A^2 + B^2$$

$\therefore \underline{(x-a)^2 + (y-b)^2 = A^2 + B^2}$

Constants to be determined by initial conditions are: $\boxed{A, B, a, b, c, d}$

This represents a circle. (4)

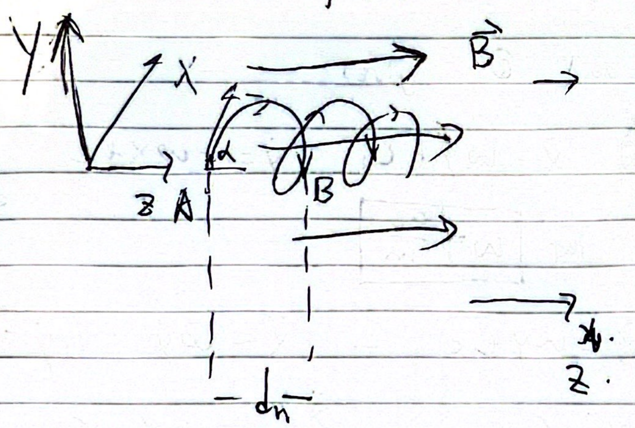
In z-direction $\underline{z = ct + d}$

This represents motion in straight line with constant velocity. (5)

Statements (4) and (5) combines to prove that the motion is a helix

(b)

Assume the particles initially makes a small angle $\underline{\alpha}$ with respect to the direction of $\vec{B}$.



The source is at point A

$\therefore$ The particles moves in circular motion as seen in the xy plane

$\therefore$ It will return to the same x, y coordinate as A after some time t_n

Magnitude of component of velocity perpendicular to $\vec{B}$ is $V\sin\alpha \approx V\alpha$

parallel to $\vec{B}$ is $V\cos\alpha \approx V$

Circular motion in xy-plane:

$$e(\alpha v)B = \frac{(\alpha v)^2}{r} m \qquad \therefore\ r = \frac{\alpha m v}{eB}$$

distance travelled per revolution (from A to B, B is a point directly above A as seen from z-axis)

is $\cancel{2\pi r = \frac{2\pi m \alpha v}{eB}}$ $\quad 2n\pi r = \frac{2n\pi m \alpha v}{eB}$

time of travel $t_n = \frac{2n\pi r}{\alpha v} = \frac{2n\pi m}{eB}$

~~diston~~ distance travelled ~~in~~ along ~~z~~ z is

$$d_n = v t_n = \boxed{\frac{2n\pi m v}{eB}}$$

∴ Particles are focused at $z = \frac{2\pi m v}{eB}$ at first, then at integral multiples of it.

(c)

Apply the equation obtained above.

$\because d_n = \frac{2n\pi m v}{eB}$ $\therefore$ $n = \left(\frac{B}{2\pi}\right)\left(\frac{d_n}{v}\right)\left(\frac{e}{m}\right)$.

$d_n = 1$ light year $\qquad v = 0.01\,c$.

$= (c)(1 \text{ year})$

$\therefore \frac{d_n}{v} = 100$ year

$$\therefore n = \left(\frac{10^{-9}\,T}{2\pi}\right)(100\times 365\times 24\times 60\times 60\,s)(1.76\times 10^{11}\,C\,kg^{-1})$$

$$= \boxed{8.8\times 10^{10} \text{ revolutions}}$$

$$X = B\sin\omega t + A(1-\cos\omega t)$$

$$Y = A\sin\omega t + B(\cos\omega t - 1)$$

$\omega = \frac{qB}{m}$ $\qquad d_n = vt$

$t_p = \frac{2\pi m}{qB} = \cancel{\ldots} = \frac{2n\pi m}{eB} v_z = \frac{2n\pi m}{eB} v$

G.3 (a) We know that $\vec{v} = (\dot{x}, \dot{y}, \dot{z})$ and $\vec{B} = (AY, AX, 0)$

$$\therefore \vec{F} = m\frac{d^2\vec{r}}{dt^2} = q\vec{v} \times \vec{B}$$

$$\therefore m(\ddot{x}, \ddot{y}, \ddot{z}) = q(\dot{x}, \dot{y}, \dot{z}) \times (AY, AX, 0)$$

$$\therefore \begin{cases} m\ddot{x} = -qAX\dot{z} \\ m\ddot{y} = qAY\dot{z} \\ m\ddot{z} = qAX\dot{x} - qAY\dot{y} \end{cases}$$

$\because$ The particle's path always make a small angle with the z-direction.

$\therefore$ we can assume that $\dot{z} = v$ and $\ddot{z} = 0$

$\therefore z = vt + c$, when $t=0$, $z=0$ $\therefore c=0$ $\therefore z = vt$

$\therefore$ The equations become.

$$\begin{cases} \ddot{x} = -\frac{qAV}{m} X & ① \\ \ddot{y} = \frac{qAV}{m} Y & ② \\ z = vt & ③ \end{cases}$$

(b) If $qA > 0$:

* Equation ① shows that the lens is focusing in the x-direction, i.e. the XZ plane. (X direction)

* Equation ② shows that the lens is defocusing in the y-direction, i.e. the YZ plane (Y direction)

$\boxed{\text{and vice versa if } qA < 0}$

(c) without loss of generality consider the XZ to be focusing (same reasoning apply for if YZ is focusing)

In the focusing plane: let $\omega^2 = \frac{|qAV|}{m}$.

$$\ddot{x} + \omega^2 x = 0.$$

$$\therefore X = X_0 \cos(\omega t + \phi)$$

when $t=0$, $x = x_0$ $\therefore \phi = 0$

$$\therefore X = X_0 \cos(\omega t)$$

When the particle first meets the z-axis it has gone through $\frac{1}{4}$ of its period of oscillation in the x-direction.

$\therefore$ time taken $t = \frac{T}{4} = \frac{1}{4}\left(\frac{2\pi}{\omega}\right) = \frac{\pi}{2}\left(\frac{1}{\omega}\right)$

$$= \frac{\pi}{2}\left(\sqrt{\frac{m}{|qA|v}}\right)$$

Distance travelled $z = vt = \frac{\pi}{2}\left(\sqrt{\frac{m}{|qA|v}}\right)(v)$

$$\therefore \boxed{z = \frac{\pi}{2}\left(\frac{mv}{|qA|}\right)^{1/2}}$$

H.O

~~$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon}$~~ Maxwell's Equations are :

$$\begin{cases} \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} & ① \\ \vec{\nabla}\cdot\vec{B} = 0 & ② \\ \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} & ③ \\ \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} & ④ \end{cases}$$

① The divergence of ~~electrical~~ electric field is proportional to the charge density ρ at that point of space

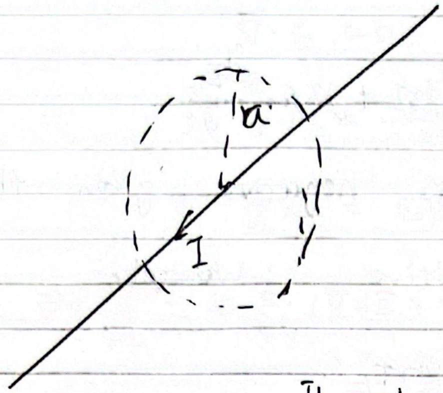
② The divergence of magnetic ~~field~~ flux density is 0 everywhere, no magnetic monopole.

③ The curl of electric field is ~~eq~~ ~~pro~~ equal to negative the rate of change of magnetic ~~field~~ flux density

④ The curl of magnetic flux density is proportional to the sum of current density and displacement current.

H.1

(a) a is the distance from wire.



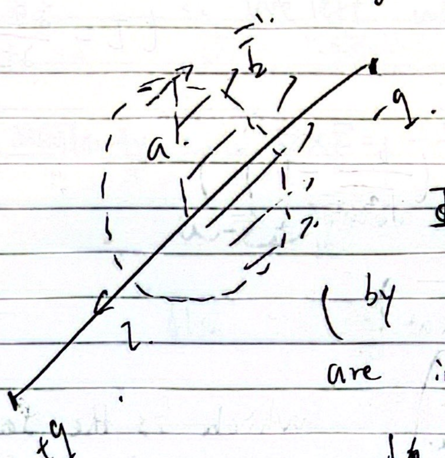
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$B(2\pi a) = \mu_0 I.$$

$$\therefore \boxed{B = \frac{\mu_0 I}{2\pi a}}$$

The identical method cannot be used to calculate the short wire because charges build up at the ends of wire ($\frac{d\rho}{dt} \neq 0$). The Ampere's Law for magnetostatics does not hold.

(b) Using modified Ampere's Law:

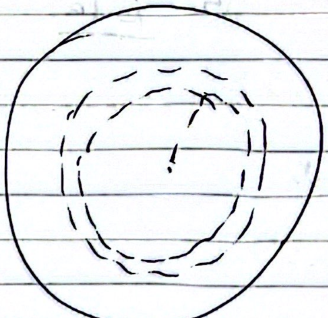


$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{\partial \Phi_E}{\partial t}$$

$$\Phi_E = \Phi_{q+} + \Phi_{q-} \qquad \therefore \Phi_{q+} = \Phi_{q-} = \int \vec{E} \cdot d\vec{s}$$

(by symmetry $\Phi_{q+} = \Phi_{q-}$ and they are in the same direction.)

~~d Φ_E = $\int \vec{E} \cdot d\vec{s}$~~ $(ds = 2\pi r dr.)$



$$\vec{E} \cdot d\vec{s} = E_\perp ds = \frac{qb}{4\pi\varepsilon_0} \frac{(2\pi r)dr}{(b^2+r^2)^{3/2}} = \frac{qb}{2\varepsilon_0} \frac{r}{(b^2+r^2)^{3/2}} dr$$

$$\therefore |\Phi_E| = 2\int \vec{E} \cdot d\vec{s}$$

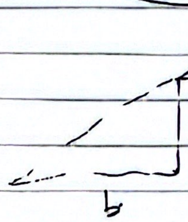
$$\therefore |\Phi_E| = 2\int \vec{E} \cdot d\vec{s}$$

$$= 2\int_0^a \frac{qb}{2\varepsilon_0} \frac{r}{(b^2+r^2)^{3/2}} dr$$

$$= 2 \cdot \frac{qb}{2\varepsilon_0} \left[-\frac{1}{\sqrt{b^2+r^2}} \right]_0^a$$

$$= 2 \times \frac{qb}{2\varepsilon_0} \left[\frac{1}{b} - \frac{1}{\sqrt{b^2+a^2}} \right]$$

$$= \frac{q}{\varepsilon_0} \left[1 - \frac{b}{\sqrt{b^2+a^2}} \right]$$



$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{(b^2+r^2)}$$

$$E_\perp = E\cos\theta$$

$$\cos\theta = \frac{b}{\sqrt{b^2+r^2}}$$

$$\therefore \cancel{E_\perp = \frac{q}{4\pi\varepsilon_0} \frac{b}{\sqrt{b^2+r^2}^3}}$$

$$E_\perp = \frac{q}{4\pi\varepsilon_0} \frac{b}{(b^2+r^2)^{3/2}}$$

∵ $\partial \Phi_E$ is increasing to the direction opposite to I.

According to $\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{\partial \Phi_E}{\partial t}$

We know $\frac{\partial \Phi_E}{\partial t}$ is negative given the direction of I is positive.

∴ Displacement current

$$I_D = \varepsilon_0 \frac{\partial \Phi_E}{\partial t} = \varepsilon_0 \frac{\partial}{\partial t}\left[-\frac{q}{\varepsilon_0}\left(1 - \frac{b}{\sqrt{b^2+a^2}}\right)\right]$$

$$= \frac{\partial q}{\partial t}\left[\frac{b}{\sqrt{b^2+a^2}} - 1\right] = I\left[\frac{b}{\sqrt{b^2+a^2}} - 1\right]$$

$$\left(I = \frac{\partial q}{\partial t}\right)$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I + I_D)$$

$$= \mu_0 \left(I + I\left(\frac{b}{\sqrt{b^2+a^2}} - 1\right)\right)$$

∴ $B(2\pi a) = \mu_0 I \left(\frac{b}{\sqrt{b^2+a^2}}\right)$

∴ $\boxed{B = \frac{\mu_0 I b}{2\pi a \sqrt{b^2+a^2}}}$ which is the same as what we found in E.1a.

H.2 (a) In vacuum the ~~math~~ Maxwell's equations become

$$\vec{\nabla}\cdot\vec{E} = 0 \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla}\times\vec{B} = \varepsilon_0\mu_0\frac{\partial \vec{E}}{\partial t}$$

$$\because \vec{\nabla}\times\vec{B} = \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$$

$$\therefore \vec{\nabla}\times\frac{\partial\vec{B}}{\partial t} = \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B}) = \varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2} \qquad \text{Also } \because \frac{\partial\vec{B}}{\partial t} = -\vec{\nabla}\times\vec{E}$$

$$\therefore -\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\therefore -(\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E}) = \varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\because \vec{\nabla}\cdot\vec{E} = 0 \quad \therefore \cancel{\nabla^2\vec{E}} = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) = 0$$

$\therefore$ We left with $\boxed{\nabla^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}}$

Similarly $\because \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ $\quad\therefore \vec{\nabla}\times\frac{\partial\vec{E}}{\partial t} = \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E}) = -\frac{\partial^2\vec{B}}{\partial t^2}$

$$\cancel{\vec{\nabla}\times\frac{\partial\vec{E}}{\partial t} = \frac{\vec{\nabla}\times(\vec{\nabla}\times\vec{B})}{\varepsilon_0\mu_0}}$$

$$\text{Also} \because \frac{\partial\vec{E}}{\partial t} = \frac{1}{\mu_0\varepsilon_0}\vec{\nabla}\times\vec{B}$$

$$\therefore \vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = -\mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\therefore -\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \frac{\partial^2\vec{B}}{\partial t^2}\mu_0\varepsilon_0$$

$$\therefore -(\vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2\vec{B}) = \frac{\partial^2\vec{B}}{\partial t^2}\mu_0\varepsilon_0$$

$$\therefore \because \vec{\nabla}\cdot\vec{B} = 0 \quad \therefore \vec{\nabla}(\vec{\nabla}\cdot\vec{B}) = 0$$

$$\therefore \cancel{\nabla^2\vec{B} = \frac{\partial^2\vec{B}}{\partial t^2}}$$

$$\therefore \boxed{\nabla^2\vec{B} = \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}}$$

(b) A plane wave is one in which the phase, and hence the disturbance, is the same at all points in a plane perpendicular to the direction of propagation of the EM wave.

Consider $\vec{E} = \hat{e}_x E_0 \sin(kz - \omega t)$ $\left(\frac{E_0}{B_0} = c = \frac{\omega}{k} \right)$

$\vec{B} = \hat{e}_y B_0 \sin(kz - \omega t)$

It is clear that they satisfy the wave equations of $\vec{E}$ and $\vec{B}$. If we take a plane perpendicular to the direction of travel of the wave (in this case z), then everywhere in that plane the $\vec{E}$ is the same and $\vec{B}$ is also the same. This is a plane wave solution. $\because \nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$, $\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$

$\therefore$ The speed of EM waves is

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$ for $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}, $\varepsilon_0 = 8.854 \times 10^{-12}$ F m^{-1}

$\therefore \boxed{c = 2.997956 \times 10^{8} \text{ m/s.}}$

(c) $\because \vec{E} = (E_0 \sin(kz - wt), 0, 0)$

Wave travelling in z - direction is:

~~$\because$ Wave vector $\vec{k} = (0, 0, k)$~~

$\because \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.

$$\therefore -\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \vec{E} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x(z) & 0 & 0 \end{vmatrix},$$

$$= \frac{\partial}{\partial z}(E_0 \sin(kz - wt))$$

~~$= kE_0 \sin(kz - wt)$~~

$$= kE_0 \cos(kz - wt)\, j = kE_0 \cos(kz - wt)\, \hat{e_y}$$

$$\therefore \frac{\partial \vec{B}}{\partial t} = -kE_0 \cos(kz - wt)\, \hat{e_y}$$

$$\therefore \vec{B} = \frac{k}{w} E_0 \sin(kz - wt)\, \hat{e_y}$$

We ignore the integration constant because we assume no steady field present in the space. $\because \frac{w}{k} = c$.

$$\therefore \vec{B} = \hat{e_y} \frac{E_0}{c} \sin(kz - wt)$$

$$= \boxed{\left(0, \frac{E_0}{c} \sin(kz - wt), 0\right)}$$

(d) The characteristic impedence

is $Z = \frac{|E|}{|H|} = \frac{|E|}{|B|/\mu_0} = \mu_0 \left|\frac{E}{B}\right|$

$= \mu_0 c = \mu_0 \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \sqrt{\frac{\mu_0^2}{\mu_0 \varepsilon_0}}$

$= \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.854 \times 10^{-12}}} = \boxed{376.7\,\Omega}$

H.3. (a)

$$\vec{E} = \hat{e_x} E_0 \sin(kz - \omega t)$$

$$\vec{B} = \hat{e_y} \frac{E_0}{c} \sin(kz - \omega t)$$

$$\vec{N} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \frac{E_0^2}{c} \sin^2(kz - \omega t)\ (\hat{e_x} \times \hat{e_y})$$

$$= \cancel{\frac{1}{\mu_0} E^2} = \boxed{\hat{e_z} \frac{E_0^2}{\mu_0 c} \sin^2(kz - \omega t)} \qquad \varepsilon_0 E^2$$

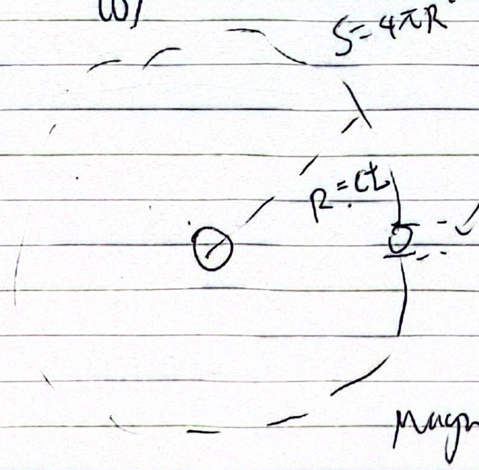
$\frac{E^2}{Z}$ ← impedance

Average value of the magnitude of poynting vector is

$$\left(\frac{\omega}{2\pi}\right) \frac{E_0^2}{\mu_0 c} \int_0^{2\pi/\omega} \sin^2(kz - \omega t)\, dt.$$

$$= \boxed{\frac{1}{2} \frac{E_0^2}{\mu_0 c}.}$$

(b).



area receive energy = A.

α = ratio of absorption = $1 - 0.3 = 0.7$

Power $P = 3.83 \times 10^{26}$ W but only P' is incident upon Earth's surface

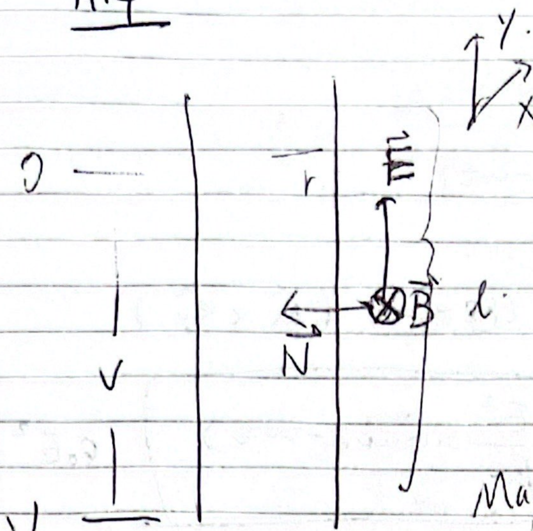
magnitude of ~~point~~ poyinting vector is the energy flows per area per time which is power per ~~to~~ area.

Thus $|N| = \frac{\alpha P'}{A}$ where $P' = \frac{P}{4\pi R^2} A$.

$$\therefore |N| = \frac{\alpha P}{4\pi R^2} = \frac{\alpha P}{4\pi c^2 t^2} = \frac{(0.7)(3.83 \times 10^{26}\,\text{W})}{(4\pi)(3 \times 10^{8})^2 (8.3 \times 60)^2}$$

$$= \boxed{955.8\ \text{J}/(\text{s} \cdot \text{m}^2)}$$

H.4



potential difference

$V = IR$ across the wire.

Neglecting the Edge Effect

Magnetic field flux density outside the wire is $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{e}_x$

Electric field is $\vec{E} = \frac{V}{l} = \frac{IR}{l}\hat{e}_y$

Poynting vector $\vec{N} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$

The ∵ $\vec{B} \perp \vec{E}$ ∴ The magnitude of $\vec{N}$

is $N = \frac{EB}{\mu_0} = \frac{1}{\mu_0}\frac{IR}{l}\frac{\mu_0 I}{2\pi r} = \boxed{\frac{I^2R}{2\pi l r}}$

its direction points to the direction into the rod

black body radiation

The energy ~~trans~~ of EM waves transfer into the rod from the exterior at a rate of $\dot{E} = NA$, where area $A = 2\pi l r$ ∴ $\dot{E} = I^2R$

We also know that the power dissipated as heat is $P = I^2R$.

∴ They are equal because of the conservation of energy. The energy lost as heat has to be ~~equal to~~ compensated by the energy of E and B fields that goes into the rod so that the energy of all particles inside the rod remains constant.