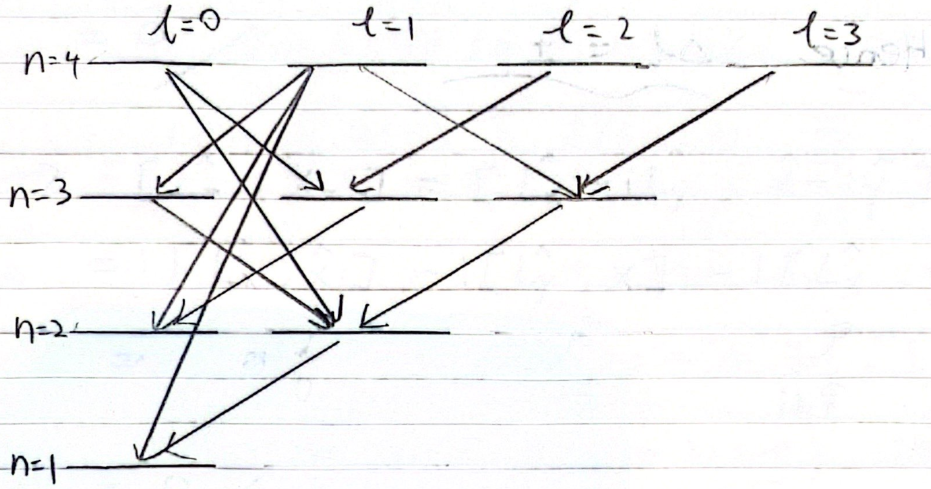
To: Felix Tennie

Further Quantum Mechanics 2 Ziyan Li

1. Selection rules for radiative transitions in the dipole approximation:

m	$\lvert m-m'\rvert \le 1$; $m=m'$ for electric field vector of emitted wave parallel to an external field; $\lvert m-m'\rvert = 1$ for photon emitted parallel to an external field;
j	$\lvert j-j'\rvert \le 1$ but $j=0$ to $j'=0$ is forbidden;
l	$\lvert l-l'\rvert = 1$;
s	$s = s'$;



The selection rules apply to the Hydrogen atom as follows:

When the ~~e~~ orbiting electron transition from a state of higher energy to a state with lower energy, a photon is emitted.

A Photon carries spin 1, hence

→ Conservation of z-component of Angular Momentum can only make the z-component of angular momentum of the ~~orbiting elect~~ Hydrogen system change

By 1, 0, or -1 (By the rule of addition of angular momentum)

So $\Delta m = +1, 0, -1$

→ Similarly the rules for adding angular momentum also ensure that the change in l is $+1$, 0, or -1.

But 0 is forbidden by ~~dipo~~ electric dipole radiation (the dipole approximation)

Hence $\Delta l = \pm 1$

2. According to QM. Q4.1

$$[\hat{L_i}, \hat{x_j}] = i\hbar\, \varepsilon_{ijk} \hat{x}_k$$

$$\hat{L}_\pm = \hat{L}_x \pm i\hat{L}_y \qquad \hat{x}_\pm = \hat{x} \pm i\hat{y}$$

$$\rightarrow [\hat{L}_+, \hat{x}_+] = [\hat{L}_x + i\hat{L}_y, \hat{x} + i\hat{y}]$$

$$= \underbrace{[\hat{L}_x, \hat{x}]}_{0} + i\underbrace{[\hat{L}_x, \hat{y}]}_{i\hbar\hat{z}} + i\underbrace{[\hat{L}_y, \hat{x}]}_{-i\hbar\hat{z}} - \underbrace{[\hat{L}_y, \hat{y}]}_{0}$$

$$= 0$$

$$\rightarrow [\hat{L}_-, \hat{x}_-] = [\hat{L}_x - i\hat{L}_y, \hat{x} - i\hat{y}]$$

$$= \underbrace{[\hat{L}_x, \hat{x}]}_{0} - i\underbrace{[\hat{L}_y, x]}_{-i\hbar\hat{z}} - i\underbrace{[\hat{L}_x, \hat{y}]}_{i\hbar\hat{z}} - \underbrace{[\hat{L}_y, \hat{y}]}_{0}$$

$$= 0$$

$$\rightarrow [\hat{L}_+, \hat{x}_-] = [\hat{L}_x + i\hat{L}_y, \hat{x} - i\hat{y}]$$

$$= \underbrace{[\hat{L}_x, \hat{x}]}_{0} + i\underbrace{[\hat{L}_y, x]}_{-i\hbar\hat{z}} - i\underbrace{[\hat{L}_x, \hat{y}]}_{i\hbar\hat{z}} + \underbrace{[\hat{L}_y, \hat{y}]}_{0}$$

$$= 2 \times i \times (-i) \times \hbar\hat{z}$$

$$= 2\hbar\hat{z}$$

$$\rightarrow -[\hat{L}_-, \hat{x}_+] = -[\hat{L}_x - i\hat{L}_y, \hat{x} + i\hat{y}].$$

$$= -\underbrace{[\hat{L}_x, \hat{x}]}_{0} + i\underbrace{[\hat{L}_y, x]}_{-i\hbar\hat{z}} - i\underbrace{[\hat{L}_x, \hat{y}]}_{i\hbar\hat{z}} - \underbrace{[\hat{L}_y, \hat{y}]}_{0}$$

$$= 2(i)(-i)\hbar\hat{z} = 2\hbar\hat{z}$$

$$\cancel{\hat{L}_z(\hat{X}_\pm|E,l,m\rangle) =}$$

$$[\hat{L}_z, \hat{X}_\pm] = [\hat{L}_z, \hat{x} \pm i\hat{y}] \cancel{= \hat{L}_z \pm}$$

$$= [\hat{L}_z, \hat{x}] \pm i[\hat{L}_z, \hat{y}] = i\hbar\hat{y} \cancel{\pm} \mp i(i)\hbar\hat{x}$$

$$= \hbar(\pm\hat{x} + i\hat{y}) = \pm\hbar\hat{X}_\pm$$

$$\hat{L}_z(\hat{X}_\pm|E,l,m\rangle) = (\hat{X}_\pm\hat{L}_z + [\hat{L}_z, \hat{X}_\pm])|E,l,m\rangle$$

$$= (\hat{X}_\pm\hat{L}_z \pm\hbar\hat{X}_\pm)|E,l,m\rangle$$

$$= \hat{X}_\pm(\hat{L}_z \pm\hbar)|E,l,m\rangle$$

$$= \hat{X}_\pm(m\pm1)\hbar|E,l,m\rangle$$

$$= (m\pm1)\hbar(\hat{X}_\pm|E,l,m\rangle)$$

Since $\hat{L}_\pm$ are the usual ladder operators

$\rightarrow$ $\hat{L}_\pm|E,l,m\rangle = \hbar\alpha_\pm(l,m)|E,l,m\pm1\rangle$

where $\boxed{\alpha_\pm(l,m) \equiv \sqrt{l(l+1)-m(m\pm1)}}$

(derived from normalisation)

$\because$ $\hat{x} = \frac{1}{2}(\hat{X}_+ + \hat{X}_-)$

Compute

$$\langle E',l',m|\hat{x}|E,l,m+1\rangle$$

$$= \frac{1}{2}\langle E',l',m|\hat{X}_+|E,l,m+1\rangle + \frac{1}{2}\langle E',l',m|\hat{X}_-|E,l,m+1\rangle$$

$= 0$ because $\hat{X}_+|E,l,m+1\rangle$ is an eigenstate of $\hat{L}_z$ with eigenvalue $m+2$, So it is Orthogonal to $|E',l',m\rangle$

not necessarily $= 0$ because $\hat{X}_-|E,l,m+1\rangle$ is an eigenstate of $\hat{L}_z$ with eigenvalue m.
$\rightarrow$ Not necessarily orthogonal to $|E',l',m\rangle$

$$= \frac{1}{2} \langle E', l', m | \hat{X}_- | E, l, m+1 \rangle$$

Similarly

$$\langle E', l', m-1 | X | E, l, m \rangle$$

$$= \frac{1}{2} \langle E', l', m-1 | \hat{X}_- | E, l, m \rangle$$

$$[\hat{L}_+, \hat{X}_-] = 2\hbar Z \rightarrow \hat{Z} = \frac{1}{2\hbar} [\hat{L}_+, \hat{X}_-]$$

$$\rightarrow \langle E', l', m | Z | E, l, m \rangle$$

$$= \frac{1}{2\hbar} \langle E', l', m | \hat{L}_+ \hat{X}_- - \hat{X}_- \hat{L}_+ | E, l, m \rangle$$

$$= \frac{1}{2\hbar} \langle E', l', m | \hat{L}_+ \hat{X}_- | E, l, m \rangle - \frac{1}{2\hbar} \langle E', l', m | \hat{X}_- \hat{L}_+ | E, l, m \rangle$$

$\hat{L}_+^\dagger = \hat{L}_-$, $\hat{L}_-^\dagger = \hat{L}_+$

$\rightarrow (\langle E', l', m | \hat{L}_+)$

$= (\hat{L}_- | E', l', m \rangle)^\dagger$

$= \hbar^* \alpha_-^* (l', m) \langle E', l', m-1 |$

$= \hbar \alpha_- (l', m) \langle E', l', m-1 |$

$\hat{L}_+ | E, l, m \rangle$

$= \hbar \alpha_+ (l, m) | E, l, m+1 \rangle$

$$= \cancel{\hbar \alpha_- (l', m) \frac{1}{2} \langle E', l', m | \hat{X}_- | E, l,}$$

$$\alpha_- (l', m) \frac{1}{2} \langle E', l', m-1 | \hat{X}_- | E, l, m \rangle$$

$$- \alpha_+ (l, m) \frac{1}{2} \langle E', l', m | \hat{X}_- | E, l, m+1 \rangle$$

$$= \alpha_- (l', m) \langle E', l', m-1 | X | E, l, m \rangle$$

$$- \alpha_+ (l, m) \langle E', l', m | X | E, l, m+1 \rangle$$

3. $\hat{H} = \frac{p^2}{2m_e} + V(\vec{x}) = \frac{1}{2m_e}(P_x^2 + P_y^2 + P_z^2) + \hat{V}(\hat{x}, \hat{y}, \hat{z})$

$$[\hat{H}, \hat{x}] = \frac{1}{2m_e}\{[\hat{P}_x^2, \hat{x}] + \underbrace{[\hat{P}_y^2, \hat{x}]}_{0} + \underbrace{[\hat{P}_z^2, \hat{x}]}_{0}\} + \underbrace{[\hat{V}, \hat{x}]}_{0}$$

$$= \frac{1}{2m_e}\hat{P}_x\underbrace{[\hat{P}_x, \hat{x}]}_{-i\hbar} + \frac{1}{2m_e}\underbrace{[\hat{P}_x, \hat{x}]}_{-i\hbar}\hat{P}_x$$

$$= \frac{1}{2m_e}(-2i\hbar)\hat{P}_x = -\frac{i\hbar\hat{P}_x}{m_e} = \frac{\hbar\hat{P}_x}{im_e}$$

$$\rightarrow [\hat{x}, [\hat{H}, \hat{x}]] = [\hat{x}, \frac{\hbar\hat{P}_x}{im_e}] = \frac{\hbar}{im_e}\underbrace{[\hat{x}, \hat{P}_x]}_{i\hbar}$$

$$= \frac{\hbar}{im_e}(i\hbar) = \frac{\hbar^2}{m_e}$$

~~Consider~~

$\cancel{\frac{\hbar^2}{m_e}\langle k | E_k, E}$

Consider $\langle k | \frac{\hbar^2}{m_e} | k \rangle = \frac{\hbar^2}{m_e}\langle k | k \rangle$

$$= \frac{\hbar^2}{m_e} = \langle k | [\hat{x}, [\hat{H}, \hat{x}]] | k \rangle$$

$$= \langle k | \hat{x}[\hat{H}, \hat{x}] - [\hat{H}, \hat{x}]\hat{x} | k \rangle$$

$$= \langle k | \hat{x}\hat{H}\hat{x} - \hat{x}^2\hat{H} - \hat{H}\hat{x}^2 + \hat{x}\hat{H}\hat{x} | k \rangle$$

$$= 2\langle k | \hat{x}\hat{H}\hat{x} | k \rangle - \langle k | \hat{x}^2\hat{H} | k \rangle - \langle k | \hat{H}\hat{x}^2 | k \rangle$$

$\because$ $\hat{H}|k\rangle = E_k|k\rangle$, and $\because$ $\hat{H}$, $\hat{x}$ are Hermitian

$\therefore$ $\langle k | \hat{H} = \langle k | E_k$

$$\rightarrow \frac{\hbar^2}{m_e} = 2\langle k | \hat{x}\hat{H}\hat{x} | k \rangle - 2E_k\langle k | \hat{x}^2 | k \rangle$$

Let $|q\rangle = \hat{x}|k\rangle$ and expand $|q\rangle$ with respect to the energy eigenbasis as

$$\hat{x}|k\rangle = |q\rangle = \sum_n C_n |n\rangle \quad \text{with} \quad C_n = \langle n|q\rangle = \langle n|\hat{x}|k\rangle$$

$$\frac{\hbar^2}{m_e} = 2\langle q|\hat{H}|q\rangle - 2E_k\langle q|q\rangle$$

$$\rightarrow \frac{\hbar^2}{2m_e} = \sum_{m,n} C_m^* C_n \langle m|\hat{H}|n\rangle - E_k \sum_{m,n} C_m^* C_n \underbrace{\langle m|n\rangle}_{\delta_{mn}}$$

$$= \sum_{m,n} E_n C_m^* C_n \delta_{mn} - E_k C_m^* C_n \delta_{mn}$$

$$= \sum_n (E_n - E_k) C_n^* C_n = \sum_n (E_n - E_k)|C_n|^2$$

When $n=k$, $E_n = E_k$ $\rightarrow$ No contribution to the sum

$\rightarrow$ We can remove the term $n=k$

$\rightarrow$ $\sum_n$ change to $\sum_{n\neq k}$

$\because$ $|C_n|^2 = |\langle n|\hat{x}|k\rangle|^2$

$$\rightarrow \sum_{n\neq k} |\langle n|\hat{x}|k\rangle|^2 (E_n - E_k) = \frac{\hbar^2}{2m_e}$$

QED.

Oscillator strength : $f_{kn} \equiv \frac{2m_e}{\hbar^2}(E_n - E_k)|\langle n|\hat{x}|k\rangle|^2$

then $$\sum_n f_{kn} = \frac{2m_e}{\hbar^2}\sum_n (E_n - E_k)|\langle n|\hat{x}|k\rangle|^2$$

$$= \frac{2m_e}{\hbar^2}\sum_{n\neq k} |\langle n|\hat{x}|k\rangle|^2 (E_n - E_k)$$

$$= \frac{2m_e}{\hbar^2}\cdot\frac{\hbar^2}{2m_e} = 1$$

QED.

The oscillator strength is a dimensionless ~~&~~ quantity that expresses the probability of absorption or emission of electromagnetic radiation in transitions between energy levels of an atom or molecule.

$\because$ the term $|\langle n|x|k\rangle|^2$ is ~~per~~ proportional to the allowed radiative transition rates, and is also proportional to the oscillator strength

Hence the larger the oscillator strength the more likely the transition will occur.

(6)

4. According to the selection rules for radiative transitions

$$\langle n'l'm'| z | n l m \rangle \neq 0 \quad \text{iff}$$

$$|l'-l| = 1 \quad \text{and} \quad m' = m$$

$\rightarrow$ $\langle 100 | z | 210 \rangle$ and $\langle 100 | z | 310 \rangle$ are non-zero

$$\langle n'l'm'| x | n l m \rangle \neq 0 \quad \text{iff}$$

$$|l'-l| = 1 \quad \text{and} \quad |m'-m| = 1$$

$\rightarrow$ $\langle 100 | x | 211 \rangle$ is non-zero

all other matrix elements are zero !

5 Consider $\underbrace{\langle r, \theta, \phi | n, l, m \rangle}_{\text{full wavefunction}} = \underbrace{u_n^l(r)}_{\text{radial wavefunction}} \underbrace{Y_l^m(\theta, \phi)}_{\text{angular wavefunction} \Rightarrow \text{the spherical harmonics.}}$

let it be $I_r(n_1, l_1; n_2, l_2)$

$$\langle 100 | \underbrace{z}_{r\cos\theta} | 210 \rangle = \underbrace{\int_0^\infty dr\, r^3 u_1^0(r) u_2^1(r)}_{I_r(n_1,l_1;n_2,l_2)} \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin\theta \times$$

$$(Y_0^0(\theta,\phi))(\cos\theta)(Y_1^0(\theta,\phi))$$

$$= I_r(1,0;2,1) \times \underbrace{\left(\frac{1}{\sqrt{4\pi}}\right)\left(\frac{\sqrt{6}}{\sqrt{8\pi}}\right)}_{\frac{\sqrt{3}}{4\pi}} \times \underbrace{\int_0^{2\pi} d\phi}_{2\pi} \underbrace{\int_0^\pi d\theta \sin\theta \cos^2\theta}_{2/3}$$

$$= I_r(1,0;2,1) \times \frac{\sqrt{3}}{3}$$

$$\langle 100 | x | 211 \rangle = \int_0^\infty dr\, r^3 u_1^0(r) u_2^1(r) \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin\theta \times$$

$$(Y_0^0(\theta,\phi))(\sin\theta\cos\phi)(Y_1^1(\theta,\phi))$$

$$= I(1,0;2,1) \times \frac{1}{\sqrt{4\pi}} \times \left(-\frac{\sqrt{3}}{\sqrt{8\pi}}\right) \underbrace{\int_0^{2\pi} d\phi \cos\phi\, e^{i\phi}}_{\pi} \underbrace{\int_0^\pi d\theta \sin^3\theta}_{\frac{4}{3}}$$

$$= -\frac{\sqrt{3}}{\sqrt{2}} \frac{1}{4\pi} \cdot \pi \cdot \frac{4}{3} = -\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{3} \times I_r(1,0;2,1)$$

$$\langle 100 | y | 211 \rangle = \int_0^\infty dr\, r^3 u_1^0(r) u_2^1(r) \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin\theta \times$$

$$(Y_0^0(\theta,\phi))(\sin\theta\sin\phi)(Y_1^1(\theta,\phi))$$

$$= I_r(1,0;2,1) \times \sqrt{\frac{1}{4\pi}} \times \left(-\sqrt{\frac{3}{8\pi}}\right) \underbrace{\int_0^{2\pi} d\phi \sin\phi\, e^{+i\phi}}_{-i\pi} \underbrace{\int_0^{\pi} d\theta \sin^3\theta}_{\frac{4}{3}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}} \times \frac{1}{4\pi} \times (-i\pi) \times \frac{4}{3} \quad = -\frac{i}{\sqrt{2}} \frac{\sqrt{3}}{3} I_r(1,0;2,1)$$

$$\to \langle 100|(x-iy)|211\rangle = \langle 100|x|211\rangle - i\langle 100|y|211\rangle$$

$$= -\frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{3} I_r(1,0;2,1)\right) - \frac{i(-i)}{\sqrt{2}}\left(\frac{\sqrt{3}}{3} I_r(1,0;2,1)\right)$$

$$= -\sqrt{2}\left(\frac{\sqrt{3}}{3} I_r(1,0;2,1)\right)$$

$$= -\sqrt{2}\,\langle 100|z|210\rangle$$ QED

$$\langle 100|x|2,1,-1\rangle = \int_0^\infty dr\, r^3 u_1^0(r) u_2^1(r) \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \times$$

$$(Y_0^0(\theta,\phi))(\sin\theta\sin\phi)(Y_1^{-1}(\theta,\phi))$$

$$= I_r(1,0;2,1) \int_0^{2\pi} d\phi\, \cancel{\sin\phi}\cos\phi\, e^{-i\phi} \int_0^{\pi} d\theta \sin^3\theta \times \sqrt{\frac{1}{4\pi}} \times \sqrt{\frac{3}{8\pi}}.$$

Similarly

$$\langle 100|y|21-1\rangle = I_r(1,0;2,1) \int_0^{2\pi} d\phi\, \cancel{\cos\phi}\sin\phi\, e^{-i\phi} \int_0^{\pi} d\theta \sin^3\theta \times \sqrt{\frac{1}{4\pi}} \times \sqrt{\frac{3}{8\pi}}$$

$$\therefore \langle 100|x-iy|21-1\rangle = \text{const} \times \int_0^{2\pi} d\phi\, \cancel{(\sin\phi - i\cos\phi)}(\cos\phi - i\sin\phi)\, e^{-i\phi}$$

$$\propto \frac{1}{i}\int_0^{2\pi} d\phi\, e^{-2i\phi} \quad \propto \quad -\frac{1}{2i}\left[e^{-2i\phi}\right]_0^{2\pi} = 0$$

$$\to \langle 100|x-iy|21-1\rangle = 0$$ QED.

Write down

$$\langle 100|x+iy|21{-1}\rangle = \sqrt{2}\langle 100|z|210\rangle$$

$$\langle 100|x+iy|211\rangle = 0$$

then consider $|\psi\rangle \equiv \frac{1}{\sqrt{2}}\left(|211\rangle - |21{-1}\rangle\right)$

$$\langle 100|x|\psi\rangle = \frac{1}{2}\langle 100|x+iy+x-iy|\psi\rangle$$

$$= \frac{1}{2\sqrt{2}}\underbrace{\langle 100|x+iy|211\rangle}_{0} + \frac{1}{2\sqrt{2}}\underbrace{\langle 100|x\mp iy|211\rangle}_{-\sqrt{2}\langle 100|z|210\rangle}$$

$$-\frac{1}{2\sqrt{2}}\underbrace{\langle 100|x+iy|21{-1}\rangle}_{\sqrt{2}\langle 100|z|210\rangle} - \frac{1}{2\sqrt{2}}\underbrace{\langle 100|x-iy|21{-1}\rangle}_{0}$$

$$= \left(-\frac{1}{2}-\frac{1}{2}\right)\langle 100|z|210\rangle = -\langle 100|z|210\rangle$$

QED

$$\rightarrow \left|\langle 100|x|\psi\rangle\right|^2 = \left|\langle 100|z|210\rangle\right|^2$$

$\because$ The transition rate for transition from state $|i\rangle$ to state $|f\rangle$ is proportional to $|\langle f|\underline{\hat{n}}\cdot\underline{\hat{x}}|i\rangle|^2$

where $\underline{\hat{n}}\cdot\underline{\hat{x}}$ is the component of the position vector operator parallel to the electric field of the radiation that is being emitted as a result of the transition.

Hence, the physical significance of the result is that the rate of ~~e radiation~~ emission ~~in the~~ of radiation polarised in the x direction as a result of a transition from state $|\psi\rangle$ to state $|100\rangle$, is the same as the rate of emission of radiation polarised in the z direction as a result of a transition from state $|210\rangle$ to state $|100\rangle$.

Also, this result tells us that $|\psi\rangle$ is an ~~sta~~ eigenstate of $\hat{L_x}$ with eigenvalue 0, just like $|210\rangle$ is an eigenstate of $\hat{L_z}$ with eigenvalue 0 the

6. $\hat{x}_+ = \hat{x} + i\hat{y}$ $\hat{x}_- = \hat{x} - i\hat{y}$

$$\cancel{\hat{J}_z(\hat{x}_\pm |n\ell m\rangle)} \quad [\hat{J}_z, \hat{x}_\pm] = [\hat{J}_z, \hat{x}] \pm i[\hat{J}_z, \hat{y}]$$

$$= i\hbar y \pm i(-i\hbar x) = (\pm\hat{x} + i\hat{y})\hbar$$

$$= \pm\hat{x}_\pm\hbar$$

$$\hat{J}_z(\hat{x}_\pm |n\ell m\rangle) = \hat{x}_\pm \hat{J}_z |n\ell m\rangle + [\hat{J}_z, \hat{x}_\pm]|n\ell m\rangle$$

$$= m\,\hat{x}_\pm |n\ell m\rangle \pm \hat{x}_\pm |n\ell m\rangle$$

$$= (m \pm 1)(\hat{x}_\pm |n\ell m\rangle)$$

$\rightarrow$ $\hat{x}_\pm |n\ell m\rangle$ is an eigenvector of $\hat{J}_z$ with eigenvalue $m \pm 1$

$\hat{J}_z$ is hermitian $\rightarrow$ ~~eigenve~~ eigenstates with different eigenvalues of $\hat{J}_z$ are orthogonal

$\langle n'\ell' m' | \hat{x}_+ | n\ell m\rangle = 0$ unless $m' = m+1$ because only in this case is $\hat{x}_+|n\ell m\rangle$ an eigenstate of m'

$\langle n'\ell' m' | \hat{x}_- | n\ell m\rangle = 0$ unless $m' = m-1$ because only in this case is $\hat{x}_-|n\ell m\rangle$ an eigenstate of m'

These selection rules have the following meanings.

$\rightarrow \langle n'l'm' | \hat{x}_+ | nlm \rangle = 0$ unless $m' = m+1$

represents the transition rate in the polarising direction $\hat{x}_+ = \hat{x} + i\hat{y}$

when $\Delta m = +1$, the rate > 0 ($= 0$ otherwise)

polarisation in ~~$\hat{x} \pm i\hat{y}$~~ $x + iy$

$\rightarrow$ y lags x by $\frac{\pi}{2}$

$\rightarrow$ left-hand polarisation in $+z$ direction (photon spin -1)

$\rightarrow \langle n'l'm' | \hat{x}_- | nlm \rangle = 0$ unless $m' = m-1$

represents the transition rate in the polarising direction $\hat{x}_- = \hat{x} - i\hat{y}$

When $\Delta m = -1$, the rate > 0 ($= 0$ otherwise)

polarisation in $x - iy$

$\rightarrow$ y leads x by $\frac{\pi}{2}$

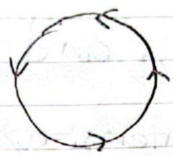
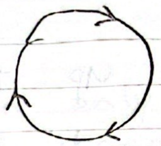
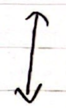
$\rightarrow$ right-hand polarisation in $+z$ direction (photon spin $+1$)

~~Hence we will~~

the presence of magnetic field in the z-direction

Because of Zeeman Effect, the radiation derived from $\Delta m = 1$ and $\Delta m = -1$ will have different energies, and thus different frequencies. Hence ~~I we~~ the spectrometer will see two separate beams of light (photons) of different frequencies, one left circularly polarised and the other right circularly polarised.

Linearly polarised photons, however, can be observed from an atom that's in a magnetic field, as long as we place the spectro-meter such that it looks along a direct-ion that is perpendicular to the $\hat{z}$ direction (say the $\hat{x}$-direction).

	observe z	observe x
$\Delta m = +1$		
		$\|+,z\rangle \quad \|-,z\rangle = \frac{1}{\sqrt{2}}(\|+,x\rangle - \|-,x\rangle)$
$\Delta m = -1$		
		$\|+,z\rangle = \frac{1}{\sqrt{2}}(\|+,x\rangle + \|-,x\rangle)$
$\Delta m = 0$	No radiation.	

What used to be circularly polarised radiation when observed from z turns to linearly polarised radiation when observed from x.
Because of Zeeman effect, radiation derived from $\Delta m = +1, 0, -1$ all have different frenqu-encies so that do not superpose

→ Three different linearly polarised light beam (photons) can be ~~oberser~~ observed.

When there is no magnetic field, there is no Zeeman effect.

→ left and right polarised radiation have the same ~~frequencies~~ frequency → they add up.

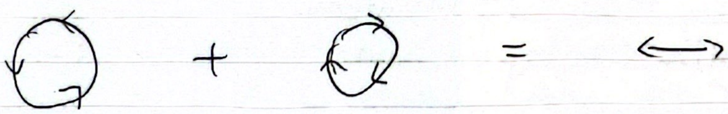
$\cancel{|\langle n' l' m' | \hat{x}_+ | n l}$

$$|\langle n' l' m+1 | \hat{x}_+ | n l m \rangle|^2 = |\langle n' l' m-1 | \hat{x}_- | n l m \rangle|^2$$

→ Same transition rate for radiations with right and left polarisations

→ Same intensity (amplitude) for them

→ They add up to form linearly polarised radiation when ~~the~~ observed from the z- direction.



→ The circularly polarised photons will no longer be observed.

→ The ~~statement~~ argument is bogus.

	1	2	3	4	5	6
	100	100	90	100	100	100

there

1

<u>FQM2</u>

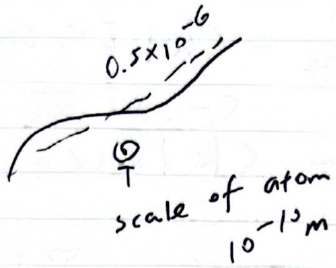
$$H_{int}^{dipole} = e\vec{d}\cdot\vec{F}$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(x)$$

$$\downarrow$$

$$\hat{p} \to \hat{p} - e\hat{A}$$

canonical momentum



Dipole approximation: on the scale of the atom, A doesn't vary.

$$\hat{H} = \frac{(\hat{p} - e\hat{A})^2}{2m} + eU + V(x)$$

Selection Rules only valid in the dipole regime.

Selection Rules for the Quadrupole term will be different, but those terms are much smaller than dipole terms.

photons can only carry away angular momentum, it cannot carry away ~~intrinc~~ intrinsic spin

so $\Delta S = 0$

$$|\psi\rangle = |space\rangle|spin\rangle$$

$$\langle\psi_{in}|\underline{X}|\psi_{out}\rangle = \langle spin_{in}|\langle space\,in|\underline{x}|space_{out}\rangle|spin_{out}\rangle$$

$\therefore \langle spin\,in\rangle = \langle spin\,out\rangle \to \Delta S = 0$

$|spin\rangle$ is ~~unto~~ untouched by the $\underline{x}$ operator

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~~The~~ The oscillator strength scale with the difference in energy and also the transition probability

→ It is also an indication of how likely the transition will occur, but since it can be negative, it cannot be the probability itself

3. $\frac{\hbar^2}{2m_e} = \langle k | [x, [H, x]] | k \rangle$

$= \langle k | xHx + xHx - Hxx - xxH | k \rangle$

$\Rightarrow \mathbb{1} = \sum_n |n\rangle\langle n|$

$= \sum_n 2E_n |\langle n|x|k\rangle|^2 - E_k |\langle n|x|k\rangle|^2 - E_k |\langle n|x|k\rangle|^2.$

$= \ldots$

~~4~~. 5.

$|100\rangle$ is spherically symmetrical state

so ~~x z~~ resulting matrix element is independent of the chosen axis of symmetry x or z transition to eigenstates $|L_x\rangle$ or $|L_z\rangle$

6.

$$|\varphi\rangle = \frac{1}{\sqrt{2}}|x,+\rangle + \frac{1}{\sqrt{2}}|x,-\rangle$$

$$\downarrow$$

$$|z,+\rangle$$