EM Set 1. Ziyan Li

A.0.

The electric field E at a point $\vec{r}$, generated by a distribution of charges q_i, is equal to the Force per unit charge q that a small test charge would experience if it was at r.

$$\vec{E}(\vec{r}) = \frac{\vec{F}(\vec{r})}{q}.$$

The electric potential V at point $\vec{r}$ is the energy W per unit charge q to move a small test charge q from a reference point $\vec{r_0}$ to $\vec{r}$.

$$\vec{E} = -\nabla V$$
$$\vec{F} = -\nabla W$$

$$V(\vec{r}) = \frac{W(\vec{r})}{q}.$$

$$\because W(\vec{r}) = -\int_{r_0}^{r} F(\vec{r}')\, d\vec{r}'$$

$$\therefore V(\vec{r}) = \frac{W(\vec{r})}{q} = -\int_{\vec{r_0}}^{\vec{r}} \frac{F(\vec{r}')}{q}\, d\vec{r}'$$

$$= -\int_{\vec{r_0}}^{\vec{r}} E(\vec{r}')\, d\vec{r}'$$

$$\therefore E(\vec{r}) = -\nabla V(\vec{r})$$

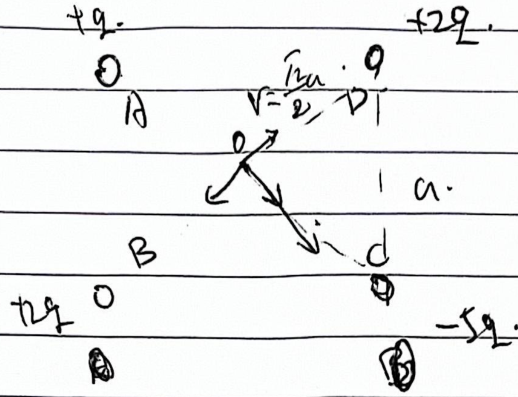
The Principle of superposition states that the net electric field is the vectorial sum of all of its components, whereas the net potential is the scalar sum of its components

Namely

$$E(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{(\vec{r}-\vec{r_i})^2} \frac{\vec{r}-\vec{r_i}}{|r-r_i|}$$

$$V(r) = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{|r-r_i|}.$$

A.1.



let · $OA=OB=OC=OD=r=\frac{\sqrt{2}}{2}a$. $\therefore r^2=\frac{1}{2}a^2$.

$\vec{E_O}=\vec{E_A}+\vec{E_B}+\vec{E_C}+\vec{E_D}$

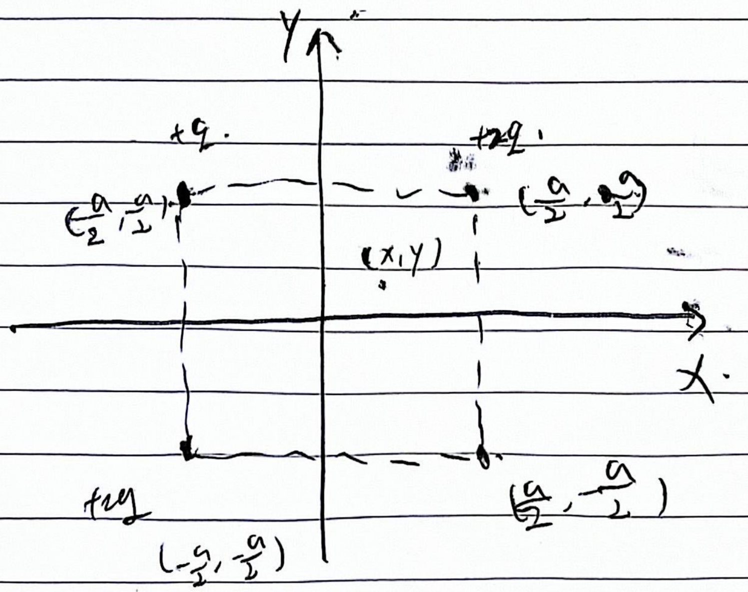
$\because Q_B=Q_B \quad \therefore \vec{E_B}+\vec{E_D}=0$

$\therefore \vec{E_O}=\vec{E_C}+\vec{E_A}=\frac{1}{4\pi\varepsilon_0}\frac{(5q)}{r^2}+\frac{q}{4\pi\varepsilon_0 r^2}=\frac{6q}{4\pi\varepsilon_0 r^2}$

$=\frac{6q}{4\pi\varepsilon_0(\frac{1}{2}a^2)}=\boxed{\frac{12q}{4\pi\varepsilon_0 a^2}}$ $\boxed{\text{towards } C}$

$V=V_A+V_B+V_C+V_D$

$=\frac{1}{4\pi\varepsilon_0 r^2}(q+2q+2q-5q)=\boxed{0}$



$$\therefore V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{|r - r_i|}$$

$$= \frac{1}{4\pi\varepsilon_0} \Big\{ q \left[(x+\tfrac{a}{2})^2 + (y-\tfrac{a}{2})^2 \right]^{-1/2} + 2q \left[(x+\tfrac{a}{2})^2 + (y+\tfrac{a}{2})^2 \right]^{-1/2}$$

$$-5q \left[(x-\tfrac{a}{2})^2 + (y+\tfrac{a}{2})^2 \right]^{-1/2} + 2q \left[(x-\tfrac{a}{2})^2 + (y-\tfrac{a}{2})^2 \right]^{-1/2} \Big\}$$

$$-\frac{\partial V}{\partial x} = \frac{1}{4\pi\varepsilon_0} \Big\{ q(\tfrac{1}{2}) \left[(x+\tfrac{a}{2})^2 + (y-\tfrac{a}{2})^2 \right]^{-3/2} (2)(x+\tfrac{a}{2})$$

$$+2q(\tfrac{1}{2}) \left[(x+\tfrac{a}{2})^2 + (y+\tfrac{a}{2})^2 \right]^{-3/2} (2)(x+\tfrac{a}{2})$$

$$-5q(\tfrac{1}{2}) \left[(x-\tfrac{a}{2})^2 + (y+\tfrac{a}{2})^2 \right]^{-3/2} (2)(x-\tfrac{a}{2})$$

$$+2q(\tfrac{1}{2}) \left[(x-\tfrac{a}{2})^2 + (y-\tfrac{a}{2})^2 \right]^{-3/2} (2)(x-\tfrac{a}{2}) \Big\}$$

$$\therefore -\frac{\partial V}{\partial x}\Big|_{x=0,y=0} = \frac{1}{4\pi\varepsilon_0} \Big\{ (q)\left(\frac{2\sqrt{2}}{a^3}\right)\left(\frac{1}{2}a\right)$$

$$+ 0 - 5(q)\left(\frac{2\sqrt{2}}{a^3}\right)\left(-\frac{a}{2}\right) \Big\}$$

$$= \frac{6\sqrt{2}}{4\pi\varepsilon_0 a^2}$$

Similarly we have

$$-\frac{\partial V}{\partial y}\Big|_{x=0,y=0} = -\frac{6\sqrt{2}}{4\pi\varepsilon_0 a^2}$$

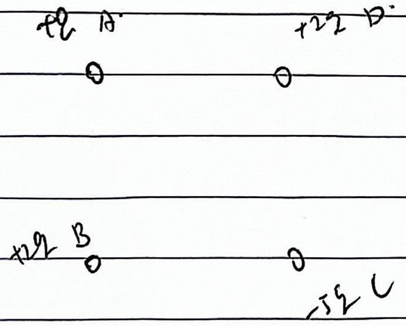
$\therefore -\nabla V = -\frac{\partial V}{\partial x} i - \frac{\partial V}{\partial y} j$

$= \boxed{\frac{6\sqrt{2}q}{4\pi\varepsilon_0 a^2} i - \frac{6\sqrt{2}q}{4\pi\varepsilon_0 a^2} j}$

which is equivalent to $\vec{E}_0$, which is $\frac{12q}{4\pi\varepsilon_0 a^2}$ towards C.

(b)

$U = \frac{1}{8\pi\varepsilon_0} \sum_i q_i \sum_{i\neq j} \frac{q_j}{r_{ji}} = \frac{1}{2}\sum_i q_i V_i$.



$V_A = \frac{1}{4\pi\varepsilon_0}\left\{ \frac{+2q}{a} + \frac{2q}{a} - \frac{5q}{\sqrt{2}a} \right\}$

$= \frac{1}{4\pi\varepsilon_0}\left\{ \left(4 - \frac{5}{\sqrt{2}}\right)\frac{q}{a} \right\}$

$= \frac{4 - \frac{5}{2}\sqrt{2}}{4\pi\varepsilon_0} \frac{q}{a}$

$= \frac{8 - 5\sqrt{2}}{8\pi\varepsilon_0} \frac{q}{a}$.

$V_B = \left\{ +\frac{q}{a} - \frac{5q}{a} + \frac{2q}{\sqrt{2}a} \right\} \frac{1}{4\pi\varepsilon_0}$

$= \frac{1}{4\pi\varepsilon_0}(\sqrt{2} - 4)\frac{q}{a} = \frac{2\sqrt{2} - 8}{8\pi\varepsilon_0} \frac{q}{a}$.

$V_C = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{a} + \frac{2q}{a} + \frac{q}{\sqrt{2}a}\right) = \frac{8 + \sqrt{2}}{8\pi\varepsilon_0}$.

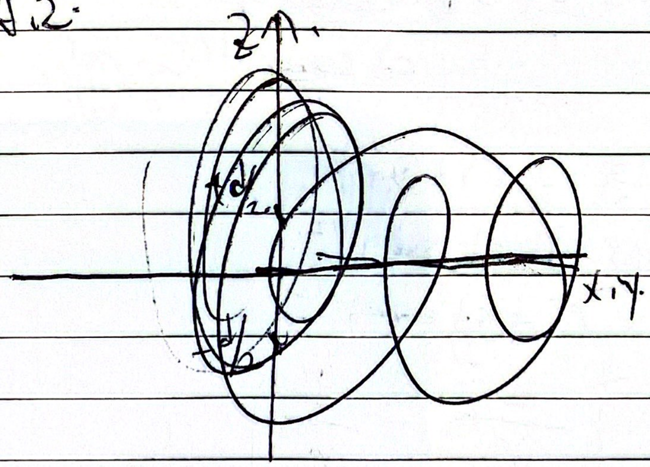
$V_D = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{a} - \frac{5q}{a} + \frac{2q}{\sqrt{2}a}\right) = \frac{2\sqrt{2} - 8}{8\pi\varepsilon_0} \frac{q}{a}$.

$$U = \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{2}\right)\left\{(8-5\sqrt{2}) + 4(2\sqrt{2}-8) - 5(8+\sqrt{2})\right\}$$

$$= \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{2}\right)\left(-64 - 2\sqrt{2}\right)$$

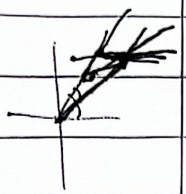
$$= \boxed{\frac{-(32+\sqrt{2})q^2}{8\pi\varepsilon_0 a}}$$

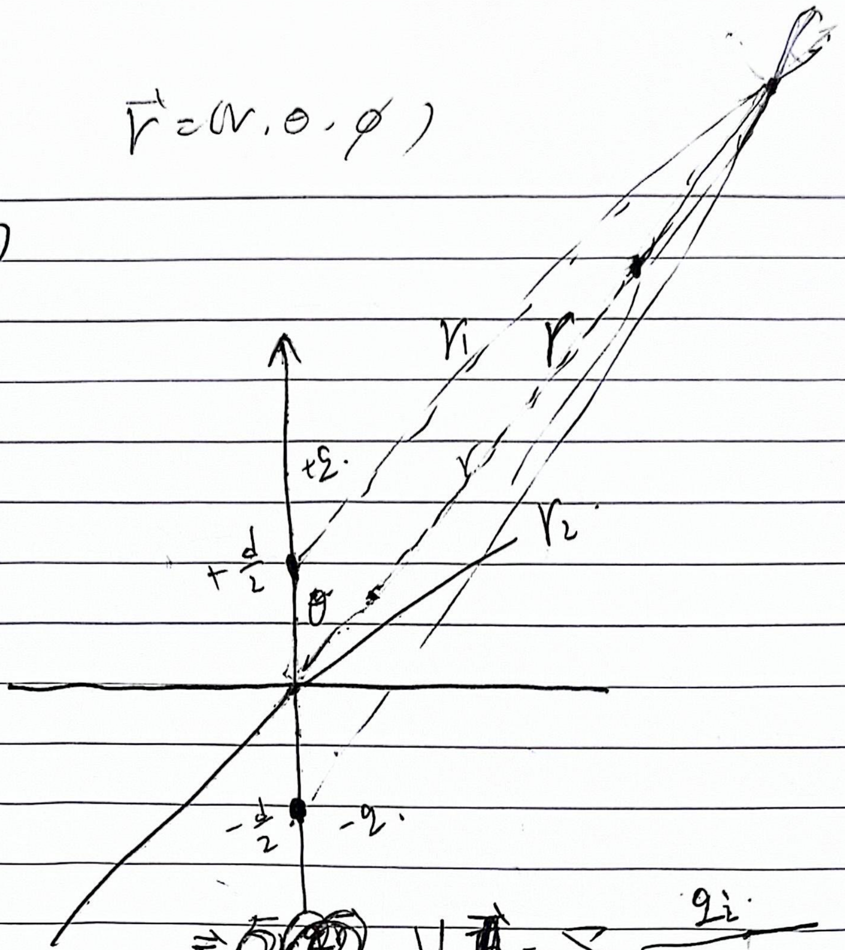
A.2.



$\vec{r} = (r, \theta, \varphi)$

(2)





$V = \sum_{i=1,2} \frac{q_i}{r_i (4\pi\varepsilon_0)}$

$r_1 = \left(r^2 + \left(\frac{d}{2}\right)^2 - r\cdot d \cancel{\cos\theta}\right)^{1/2}$

$r_2 = \left(r^2 + \left(\frac{d}{2}\right)^2 + r\cdot d \cancel{\cos\theta}\right)^{1/2}$

$\because r \gg \frac{d}{2}$;

$\therefore r_1 \approx \left(r^2 - r\cdot d \cancel{\cos\theta}\right)^{1/2}$

$r_2 \approx \left(r^2 + r\cdot d \cancel{\cos\theta}\right)^{1/2}$

$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_1} - \frac{q}{r_2}\right]$

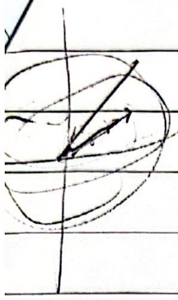
$= \frac{1}{4\pi\varepsilon_0} q \left[\left(r^2 - r\cdot d \cancel{\cos\theta}\right)^{-1/2} - \left(r^2 + r\cdot d \cancel{\cos\theta}\right)^{-1/2}\right]$

$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\left[\left[1 - \frac{r\cdot d}{r^2}\cancel{\cos\theta}\right]^{-1/2} - \left[1 + \frac{r\cdot d}{r^2}\cancel{\cos\theta}\right]^{-1/2}\right]$

$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\left[\left[1 + \frac{r\cdot d}{2r^2}\cancel{\cos\theta}\right] - \left[1 - \frac{r\cdot d}{2r}\cancel{\cos\theta}\right]\right]$

$\cancel{= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\left[\frac{r\cdot d}{r^2}\cos\theta\right]}$

$= \cancel{\frac{qd\cos\theta}{4\pi\varepsilon_0 r^2}}$



$$= \cancel{qdr\cos\theta}\ \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\left[\frac{r\cdot d}{r^2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r^3}\right)(r\cdot d)$$

$$= \frac{q(r\cdot d)}{4\pi\varepsilon_0 r^3} = \frac{q\vec{d}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$$

$\because \vec{p} = q\vec{d}$

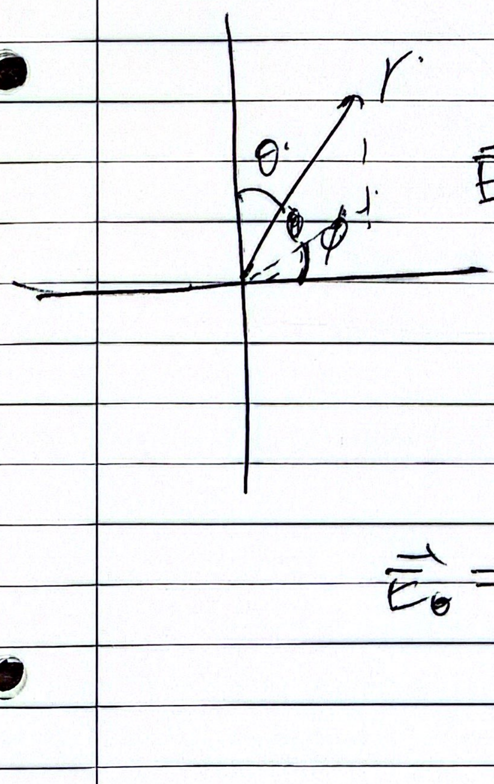
$$\therefore \boxed{V = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}}$$

$$E = -\nabla V = -\frac{\partial V}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} - \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$$

$\because p = (qd, 0, 0)$

$r = (r, \theta, \phi)$

$p\cdot r = |p||r|\cos\theta = qdr\cos\theta$



$$\vec{E_r} = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{qd\cos\theta}{4\pi\varepsilon_0} r^{-2}\right)$$

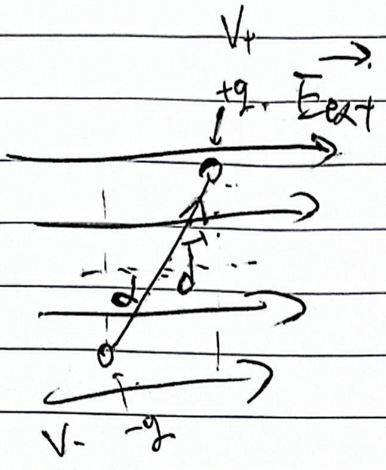
$$= \frac{2qd\cos\theta}{4\pi\varepsilon_0} r^{-3}$$

$$= \frac{2qdr\cos\theta}{4\pi\varepsilon_0 r^4} = \boxed{\frac{2\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^4}}$$

$$\vec{E_\theta} = -\frac{1}{r}\frac{\partial V}{\partial \theta} = -\frac{1}{r}\frac{qdr(-\sin\theta)}{4\pi\varepsilon_0 r^3}$$

$$= \boxed{\frac{qd\sin\theta}{4\pi\varepsilon_0 r^3}}$$

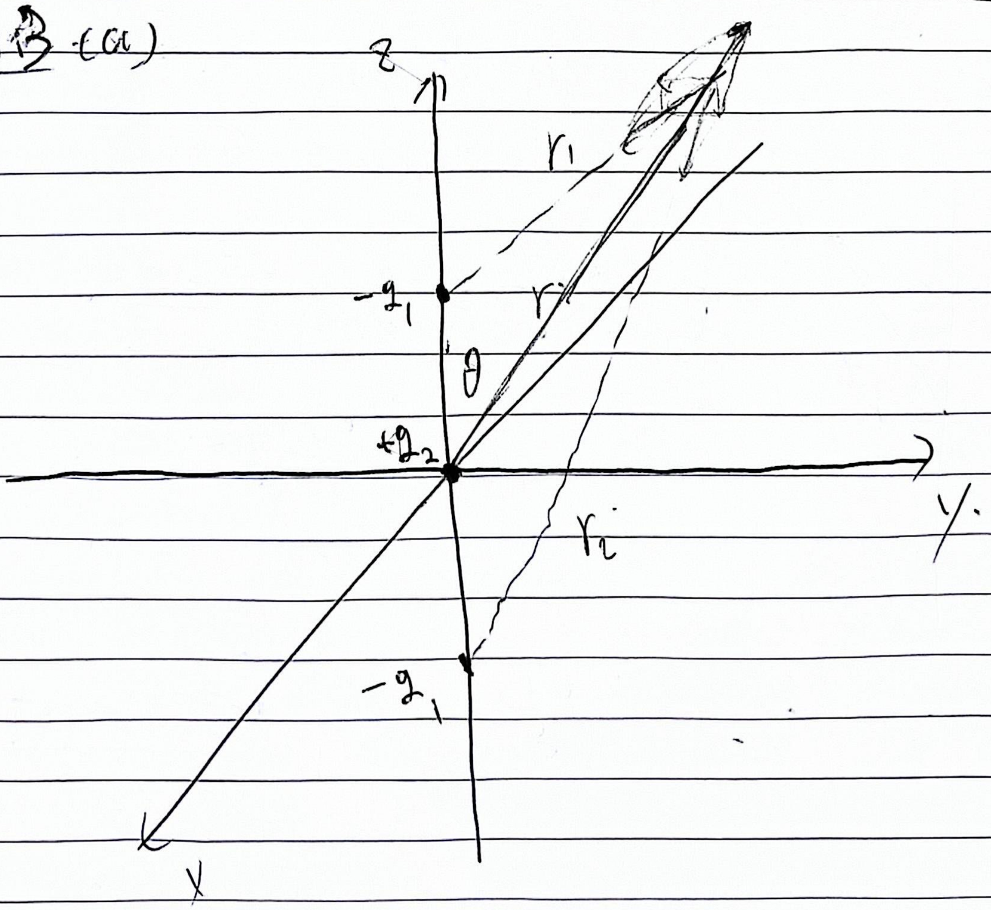
$\because \frac{\partial V}{\partial \phi} = 0 \quad \therefore \boxed{E_\phi = 0}$



$$\vec{p} = q\vec{d}$$

$$U = (+q)(V_+) + (-q)(V_-)$$
$$= q(V_+ - V_-)$$
$$V_+ - V_- = -\int_-^+ \vec{E}_{ext} \cdot d\vec{r}$$
$$= -|E_{ext}||d|\cos\alpha$$
$$\therefore U = -q|d||E_{ext}|\cos\alpha = \boxed{-|p||E_{ext}|\cos\alpha = -\vec{p}\cdot\vec{E}_{ext}}$$

AB (a)



$$r_1 = (r^2+a^2-2ra\cos\theta)^{1/2} \qquad r_2 = (r^2+a^2+2ra\cos\theta)^{1/2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\left\{\frac{q_2}{r} - \frac{q_1}{r_1} - \frac{q_1}{r_2}\right\}$$

$$\cancel{= \frac{1}{4\pi\varepsilon_0}\left\{\frac{q_2}{r} - q_1\right.}$$

$$= \frac{1}{4\pi\varepsilon_0}\left\{\frac{q_2}{r} - q_1[r^2+a^2-2ra\cos\theta]^{-1/2} - q_1[r^2+a^2+2ra\cos\theta]^{-1/2}\right\}$$

$$= \boxed{\frac{1}{4\pi\varepsilon_0}\left\{\frac{q_2}{r} - \frac{q_1}{r}\left[1+\frac{a^2}{r^2}-\frac{2a\cos\theta}{r}\right]^{-1/2} - \frac{q_1}{r}\left[1+\frac{a^2}{r^2}+\frac{2a\cos\theta}{r}\right]^{-1/2}\right\}}$$

(b) let $k = \frac{1}{4\pi\varepsilon_0}$, $x = \frac{a}{r}$

$$V = k\left\{\frac{q_2}{r} - \frac{q_1}{r}\left[(1-2\cos\theta x+x^2)^{-1/2} + (1+2\cos\theta x+x^2)^{-1/2}\right]\right\}.$$

$$\cancel{= k\left\{\frac{q_2}{r} - \frac{q_1}{r}\right[}$$

$$(1-2\cos\theta x+x^2)^{-1/2} \simeq 1 - \frac{1}{2}(-2\cos\theta x+x^2) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-2\cos\theta x+x^2)^2$$

$$= \cancel{1+\cos x}\ 1+\cos\theta x - \frac{1}{2}x^2 + \frac{3}{8}(-2\cos\theta x+x^2)^2$$

$$= 1+\cos\theta x - \frac{1}{2}x^2 + \frac{3}{2}\cos\theta x^2 + O(x^3)$$

$$= 1+\cos\theta x + (\frac{3}{2}\cos\theta - \frac{1}{2})x^2 + O(x^3)$$

$$(1+2\cos\theta x+x^2)^{-1/2} \simeq 1 - \frac{1}{2}(2\cos\theta x+x^2) + \frac{3}{8}(2\cos\theta x+x^2)^2.$$

$$= 1-\cos\theta x - \frac{1}{2}x^2 + \frac{3}{2}\cos\theta^2 x^2$$

$$= 1-\cos\theta x + (\frac{3}{2}\cos^2\theta - \frac{1}{2})x^2 + O(x^3)$$

$$\therefore V = k\left[\frac{q_2}{r} - \frac{q_1}{r}\left(1 + \cancel{\cos\theta x} + \left(\frac{3}{2}\cos\theta - \frac{1}{2}\right)x^2 + 1 - \cancel{\cos\theta x} + \left(\frac{3}{2}\cos\theta - \frac{1}{2}\right)x^2\right)\right]$$

$$= k\left[\frac{q_2}{r} - \frac{q_1}{r}\left(2 + (3\cos^2\theta - 1)x^2\right)\right].$$

$$\cancel{= k\left[\frac{q_2}{r} - \frac{2q_1}{r} - 3\cos\theta\right.}$$

$$= \frac{kq_2}{r} - \frac{2kq_1}{r} - \frac{3k\cos^2\theta\, a^2 q_1}{r^3} + \frac{ka^2 q_1}{r^3}.$$

$\frac{kq_2}{r}$ and $\frac{2kq_1}{r}$ have ~~monopol~~ monopole character, $\frac{3k\cos^2\theta\, a^2 q_1}{r^3} + \frac{ka^2 q_1}{r^3}$ have quadrupole character.

(c)

When $q_2 = 2q_1$.

$$V = -\frac{3ka^2\cos^2\theta\, q_1}{r^3} + \frac{ka^2 q_1}{r^3}$$

$$E = -\nabla V = -\frac{\partial V}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} - \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$$

$$E_r = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(-3ka^2\cos^2\theta\, q_1 (r^{-3})\right) - \frac{\partial}{\partial r}\left((ka^2 q_1)(r^{-3})\right)$$

$$= \boxed{-\frac{9ka^2\cos^2\theta\, q_1}{r^4} + \frac{3ka^2 q_1}{r^4}}$$

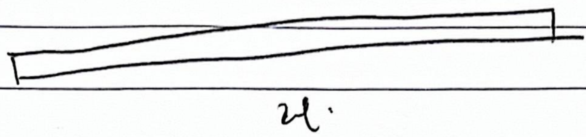
$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \cancel{\frac{3ka^2\sin\theta\, q_1}{r^4}} - \frac{ka^2 q_1}{r^4}\left(2\cos\theta(-\sin\theta)\right)$$

$$= \boxed{\frac{-ka^2 q_1 \sin(2\theta)}{r^4}}$$

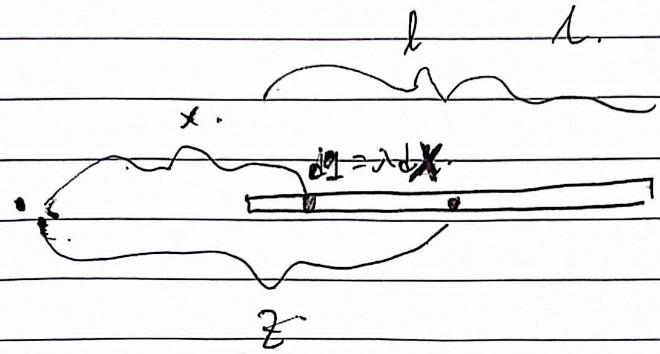
$$E_\phi = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi} = -\frac{1}{r\sin\theta}(0) = \boxed{0}.$$

$$\vec{E} = (E_r, E_\theta, E_\phi)$$

Q.4



(a)



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2}$$

$$\int dE = \frac{1}{4\pi\varepsilon_0}\int_{z-l}^{z+l}\frac{dq}{x^2}$$

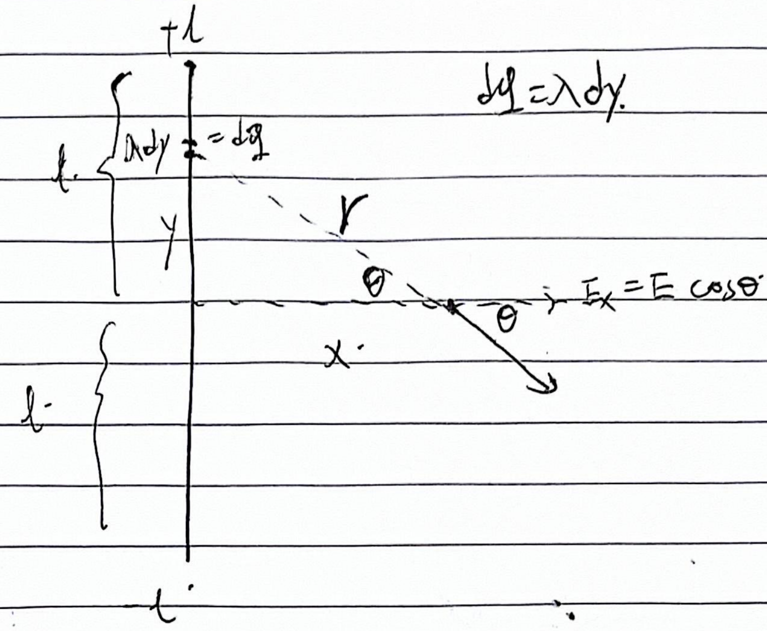
$$\therefore E = \frac{1}{4\pi\varepsilon_0}\int_{z-l}^{z+l}\frac{\lambda\, dx}{x^2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int_{z-l}^{z+l}\frac{dx}{x^2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(-\frac{1}{x}\right)\Big|_{z-l}^{z+l}$$

$$= \boxed{\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{z-l} - \frac{1}{z+l}\right)}$$

(b)



Electric field in y-direction ~~cancle~~ cancels ~~eq~~

$\therefore$ ~~only E~~ Electric fied remains only along x-direction.

$2\,(x^2+y^2)^{\frac{1}{2}}$

$$\therefore \quad dE = \frac{1}{4\pi\varepsilon_0}\,\frac{dq}{r^2}\cos\theta$$

$r = \sqrt{x^2+y^2}$, $\cos\theta = \frac{x}{r}$

$$\therefore \quad dE = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda dy}{\left(\sqrt{x^2+y^2}\right)^2}\cdot\frac{x}{\sqrt{x^2+y^2}} = \frac{\lambda x}{4\pi\varepsilon_0}\,\frac{dy}{(x^2+y^2)^{3/2}}$$

$$= \cancel{\frac{\lambda x}{4\pi\varepsilon_0}\,\frac{dy}{d\theta}}$$

$$\therefore \quad \cancel{E = \int_{-l}^{+l}} \quad E = \frac{\lambda x}{4\pi\varepsilon_0}\int_{-l}^{+l}\frac{dy}{(x^2+y^2)^{3/2}}$$

$$\cancel{\int \frac{dy}{(x^2+y^2)^{3/2}} = \int \frac{dy}{\left[x^2\left(1+\frac{y^2}{x^2}\right)\right]^{3/2}}}$$

$$\cancel{= \int \frac{dy}{x^3\left(1+\frac{y^2}{x^2}\right)^{3/2}}}$$

$$I = \int \frac{dy}{(x^2+y^2)^{3/2}} = \int \frac{dy}{\left[y^2\left(1+\frac{x^2}{y^2}\right)\right]^{3/2}} = \int \frac{\cancel{\text{sgn } y}\; dy}{y^3\left(1+\frac{x^2}{y^2}\right)^{3/2}}$$

$(\because y > 0)$

$$= \int \cancel{\text{sgn } y}\left(1+\frac{x^2}{y^2}\right)^{-3/2} y^{-3}\, dy$$

let $u = 1+\frac{x^2}{y^2}$ $\quad \frac{du}{dy} = -\frac{x^2}{y^3} = -2x^2y^{-3}$

$\therefore du = -2x^2y^{-3}dy.$ $\quad \therefore y^{-3}dy = -\frac{du}{2x^2}$

$$\therefore I = -\int \cancel{\text{sgn } y}\,(u)^{-3/2}\frac{du}{2x^2}$$

$$= \frac{-1\cancel{\text{sgn } y}}{2x^2}\int u^{-3/2}\, du = \frac{-1\cancel{\text{sgn } y}}{2x^2}\left[(-2)u^{-1/2}\right] + C.$$

$$= \frac{\cancel{\text{sgn } y}\,1}{x^2}\frac{1}{\sqrt{1+\frac{x^2}{y^2}}} + C.$$

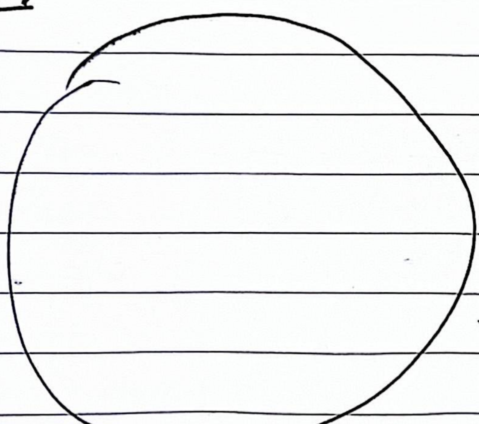
$$= \frac{y}{x^2\sqrt{x^2+y^2}} + C.$$

$$\therefore E = \frac{\lambda x}{4\pi\varepsilon_0 x^2}\left[\frac{y}{\sqrt{x^2+y^2}}\right]\Bigg|_{-l}^{l}$$

$$= \frac{\lambda x \lambda}{4\pi\varepsilon_0 x}\left[\frac{2l}{\sqrt{x^2+l^2}}\right]$$

$$= \boxed{\frac{2l\lambda}{4\pi\varepsilon_0 x\sqrt{x^2+l^2}}}$$

A3.



$r : 0 \to b$

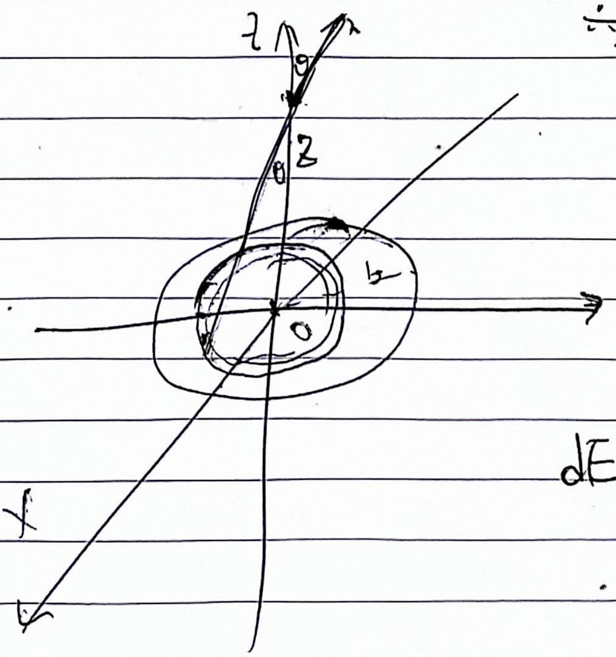
$ds = 2\pi r\, dr$

$dq = \sigma\, ds =$ ~~2π~~ $2\pi\sigma r\, dr$

All field components ~~along~~ along x and y cancel.

∴ only Net E-field is along z

~~∴ E =~~



let's consider a ring of radius r and thickness dr.

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{z\, dq}{(z^2+r^2)^{3/2}}$$

$$\therefore \quad dE = \frac{z}{4\pi\varepsilon_0} \frac{2\pi\sigma r\, dr}{(z^2+r^2)^{3/2}}$$

$$\therefore \quad E = \frac{2\pi\sigma z}{4\pi\varepsilon_0} \int_0^b \frac{r\, dr}{(z^2+r^2)^{3/2}}$$

$$= \frac{2\pi\sigma z}{4\pi\varepsilon_0} \left[-\frac{1}{\sqrt{z^2+r^2}} \right]\Bigg|_0^b$$

$$= \frac{2\pi\sigma z}{4\pi\varepsilon_0} \left[\frac{1}{z} - \frac{1}{\sqrt{z^2+b^2}} \right]$$

$$= \boxed{\frac{\sigma z}{2\varepsilon_0} \left[\frac{1}{z} - \frac{1}{\sqrt{z^2+b^2}} \right]}$$

(b).

$z \ll b \qquad \sqrt{z^2+b^2} \approx b \qquad \frac{1}{b} \approx 0$

$\therefore \quad E \approx \frac{\sigma z}{2\varepsilon_0 z} = \frac{\sigma}{2\varepsilon_0}$, which is as if the disk is a infinite plane

$z \gg b \qquad$ ~~$\sqrt{z^2+b^2} \approx \frac{1}{z} \quad z(1+\frac{b^2}{z^2})^{1/2} \approx z(1+\frac{b^2}{2z^2})$~~

~~$E = \frac{\sigma z}{2\varepsilon_0}$~~

$(z^2+b^2)^{-\frac{1}{2}} \approx \frac{1}{z}(1-\frac{b^2}{2z^2})$

$\therefore \left[\frac{1}{z} - \frac{1}{\sqrt{z^2+b^2}}\right] \approx \frac{b^2}{2z^3}$

$\therefore E = \frac{\sigma z b^2}{(2\varepsilon_0)(2z^3)} = \boxed{\frac{\sigma b^2}{4\varepsilon_0 z^2}} = \frac{\frac{Q}{\pi b^2} b^2}{4\varepsilon_0 z^2} = \boxed{\frac{Q}{4\pi\varepsilon_0 z^2}}$

$\therefore$ The disk acts like a point charge.

A.b

According to previous question

~~$|E_z| = \frac{1}{4\pi\varepsilon_0} \frac{qz}{\sqrt{z^2+a^2}}$~~

$$|E_z| = \frac{q}{4\pi\varepsilon_0} \frac{z}{(z^2+a^2)^{3/2}}$$

let $0 = \frac{d|E_z|}{dz} = \left[\frac{(z^2+a^2)^{3/2} - \frac{3}{2}(z^2+a^2)^{1/2}(2z)\cdot z}{(z^2+a^2)^3}\right] \frac{q}{4\pi\varepsilon_0}$

$\therefore (z^2+a^2)^{3/2} - \frac{3}{2}(z^2+a^2)^{1/2}(2z)\cdot z = 0$

$\therefore z^2 = \frac{(z^2+a^2)^{3/2}}{3(z^2+a^2)^{1/2}} = \frac{z^2+a^2}{3}$

$\frac{2}{3}z^2 = \frac{1}{3}a^2 \Rightarrow z^2 = \frac{a^2}{2}$

$\therefore$ At $\boxed{z = \pm\frac{a}{\sqrt{2}}}$, $|E_z|$ has maximum value.

$|E_z| = \frac{q}{4\pi\varepsilon_0} z(z^2+a^2)^{-3/2}$

~~$= \frac{q}{4\pi\varepsilon_0} z\left(\frac{1}{z^3}\right)\left(1+\frac{a^2}{z^2}\right)^{-3/2}$~~

$= \frac{qz}{4\pi\varepsilon_0 a^3}\left(1+\frac{z^2}{a^2}\right)^{-3/2}$

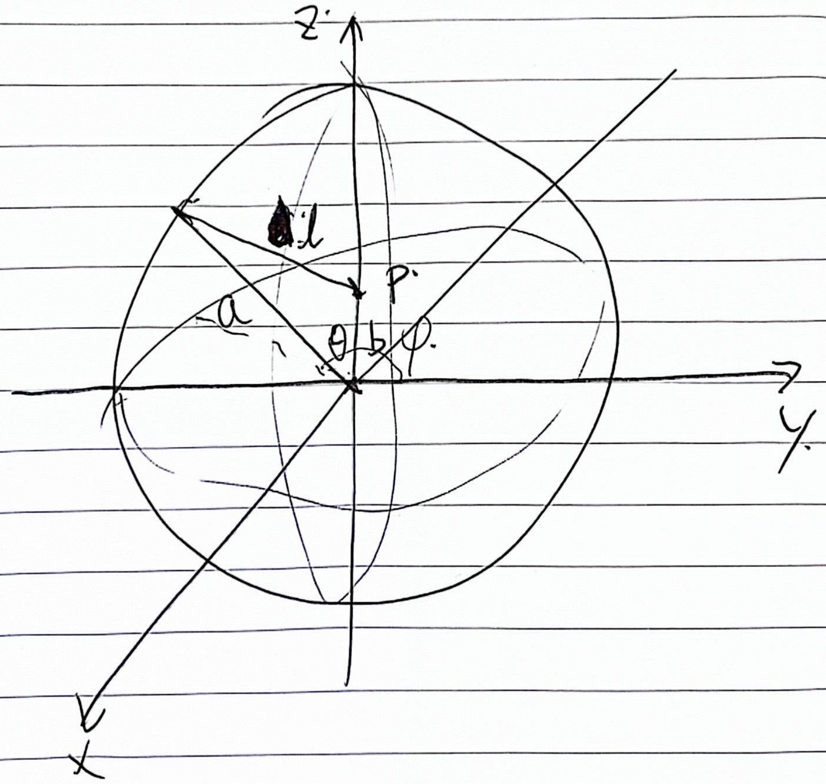
$\approx \frac{qz}{4\pi\varepsilon_0 a^3}$

$\therefore F = -e|E_z| = m\ddot{z}$ $\therefore$ $F = -\frac{eqz}{4\pi\varepsilon_0 a^3}$ $\therefore \ddot{z} = -\frac{eqz}{4\pi\varepsilon_0 a^3 m}$

$$\therefore \omega = \sqrt{\frac{eq}{4\pi\varepsilon_0 a^2 m}}$$

$$\therefore f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{eq}{4\pi\varepsilon_0 a^2 m}} = \boxed{\sqrt{\frac{eq}{16\pi^3\varepsilon_0 a^2 m}}}$$

A17.



Orienting the z-axis to contain P
we consider the ~~poten~~ potential dV produced by
a differential ~~area ds~~ charge dq over ds.

$dq = \sigma\, ds$, where $ds = (r\sin\theta\, d\phi)(r\, d\theta)$
$= r^2 \sin\theta\, d\theta\, d\phi$

$dV = \dfrac{dq}{4\pi\varepsilon_0 l}$, where $l = (a^2+b^2-2ab\cos\theta)^{1/2}$.

$r = a$.

$$\therefore \iint dV = \iint \frac{dq}{4\pi\varepsilon_0 l}$$

$$\therefore V = \frac{1}{4\pi\varepsilon_0}\iint \frac{a^2\sigma \sin\theta \, d\theta \, d\phi}{l}$$

$$= \frac{\sigma a^2}{4\pi\varepsilon_0}\int_0^{\pi}\int_0^{2\pi} \frac{\sin\theta \, d\phi \, d\theta}{(a^2+b^2-2ab\cos\theta)^{\frac{1}{2}}}$$

$$= \frac{2\pi\sigma a^2}{4\pi\varepsilon_0}\int_0^{\pi} \frac{\sin\theta \, d\theta}{(a^2+b^2-2ab\cos\theta)^{1/2}}$$

let $u = -\cos\theta$

$du = \sin\theta \, d\theta$

$$\therefore V = \frac{2\pi\sigma a^2}{4\pi\varepsilon_0}\int_{-1}^{1} \frac{du}{(a^2+b^2+2abu)^{1/2}}$$

$$= \frac{2\pi\sigma a^2}{4\pi\varepsilon_0}\left[\frac{1}{ab}(a^2+b^2+2abu)^{1/2}\right]\Big|_{-1}^{1}$$

$$= \frac{2\pi\sigma a^2}{4\pi\varepsilon_0 ab}\left[(a^2+b^2+2ab)^{1/2} - (a^2+b^2-2ab)^{1/2}\right]$$

$$= \frac{2\pi\sigma a^2}{4\pi\varepsilon_0 ab}\left[a+b-(a-b)\right]$$

$$= \frac{2\pi\sigma a^2}{4\pi\varepsilon_0 ab}(2b) = \frac{4\pi\sigma a}{4\pi\varepsilon_0} = \boxed{\frac{\sigma a}{\varepsilon_0}}$$

which is independent of b

$\therefore$ $V = \frac{a\sigma}{\varepsilon_0}$ is true at any point inside the sphere.