First-Year Electromagnetism: Problem Set 5

Hilary Term 2015, Prof LM Herz

G. Motion of Charged Particles

G.0 Background. State an expression for the force experienced by a charged particle in the presence of electric and magnetic fields.

G.1 Bainbridge mass spectrometer. In a Bainbridge mass spectrometer, ions of charge q, mass m and velocity v enter an initial velocity filter, in which they experience both a uniform E and B-field. They subsequently pass through an aperture into a chamber in which only the B-field is present. Depending on their mass, ions are potentially recorderd by a detector situated behind an exit slit.

(a) Make a sketch of the spectrometer, indicating the orientation of E- and B-fields required for operation.

(b) If the spectrometer were used with E=100 V cm^{-1} and B=0.2 T, what would be the velocity of an ion that can pass through the velocity filter?

(c) If the ion beam leaving the velocity filter has a width of 1 mm, could this machine be used to separate two isotopes of helium, He^3 and He^4?

G.2 Charged particles moving in a constant magnetic field.

(a) Show that the path of a charged particle, moving in a constant and uniform magnetic field, is, in general, a helix.

(b) Particles with a charge e and mass m are emitted with velocity v from a point source. Their directions of emission make a small angle with the direction of a uniform constant flux density B. Show that the particles are focussed to a point at a distance $2\pi mv/Be$ from their source and at integral multiples of this distance.

(c) An electron in interstellar space has a component of velocity $0.01c$ in the direction of a magnetic flux density of 10^{-9} T. How many revolutions does it make in its helical path in travelling between two points of space one light-year apart, measured along a line of force? For an electron $e/m = 1.76\times10^{11}$ C kg^{-1}. [Answer: 8.8×10^{10} treated non-relativistically]

G.3 Magnetic quadrupole lens. A particle of mass m and charge q is projected along the z-direction with speed v, in a path close to the z-axis. It enters a long magnetic quadrupole lens, within which the magnetic flux density components are given by $B_x = Ay$, $B_y = Ax$ and $B_z = 0$, where A is a constant.

(a) Write down the equations of motion for the $x-$ and $y-$components of the particle's velocity v. Assume that the magnet is free from end effects, and that the particle's path always makes a small angle with the z-direction.

(b) Show that the lens has a focusing property in one plane, but is defocusing in the other.

(c) Show that for the focusing plane, the particle first meets the z-axis after travelling a distance $\frac{\pi}{2}\left(\frac{mv}{|qA|}\right)^{1/2}$

H. Electro-Magnetic Fields and Maxwell's Equations

H.0 Background. State Maxwell's equations in the presence of free charge and electric current and comment on the information each one contains.

H.1 Displacement current.

(a) Calculate the magnetic field around a straight, long current-carrying wire, using Ampere's law. Discuss whether an identical method can be used for calculating the magnetic field generated by a short segment of such a wire.

(b) Use a modified version of Ampere's law that includes a suitable displacement current, in order to determine the magnetic field H at a distance a from a straight current-carrying wire (length $2b$) along its perpendicular bisector. Compare your result with the one you previsouly obtained from Biot-Savart's law (Question E.1a).

H.2 Electro-magnetic waves in vacuo.

(a) Show that Maxwell's equations, in a vacuum devoid of charges and currents, lead to wave equations for the electric and magnetic fields.

(b) Show that plane wave solutions may be obtained and deduce the speed of propagation of these waves.

(c) Obtain expressions for the magnetic flux density vector $\mathbf{B}$ if the electric field vector is described by $\mathbf{E} = (E_0 \sin[kz - \omega t], 0, 0)$.

(d) Determine the characteristic impedance $|E|/|H|$ of free space.

H.3 Poynting vector of an electro-magnetic wave.

(a) Calculate the Poynting vector for plane electromagnetic waves propagating in free space.

(b) The sun has a total radiative power of 3.83×10^{26} W. Electromagnetic waves emanating from the sun take 8.3 min to reach the Earth's atmosphere. Calculate the magnitude of the pointing vector on the Earth's surface when 30% of the incident sunlight is absorbed by the atmosphere.

H.4 Poynting vector for a long resistive rod. Consider a long resistive rod of length l, radius r and resistance R, which carries a current I flowing uniformly through its circular cross section. Calculate the magnitude and direction of the Poynting vector (neglecting edge effects) and relate the rate of energy transfer between the rod and its exterior to the total power dissipated in the rod.