First-Year Electromagnetism: Problem Set 4

Hilary Term 2015, Prof LM Herz

F. Electromagnetic Induction and Self-Inductance

F.0 Background. State the laws of Faraday and Lenz. What is meant by the terms "self-inductance" and "mutual inductance"?

F.1 Rectangular coil moving away from current-carrying wire. A very long straight wire carries a current I. A plane rectangular coil of high resistance, with sides of length a and b, is coplanar with the wire. One of the sides of length a is parallel to the wire and a distance D from it; the opposite side is further from the wire. The coil is moving at a speed v in its own plane and away from the wire.

(a) Find the value of the e.m.f. induced in the coil by two methods:

i. by considering the e.m.f. induced in each of the sides separately;

ii. by considering the rate of change of magnetic flux through the loop.

(b) In this problem the resistance of the coil is stated as being "high". Why is this restriction necessary in order to find the answer given?

(c) The resistance of the coil is R. Calculate the force needed to move the coil with speed v as described, and show that the mechanical power used to move it is equal to the rate at which heat is generated in the coil.

[Answers: $\mu_0 Ivab/(2\pi D(D+b))$, $(\mu_0 Ivab)^2/(4\pi^2 RD^2(D+b)^2)$]

F.2 Sliding metal rod defining the edge of a circuit. Two horizontal metal rails, separated by a distance L, run parallel to the x-axis. At $x = 0$ a resistor R is connected between the rails. A closed circuit is formed by a metal rod which slides along the rails with a constant velocity v such that its position at time t is given by $x = vt$. There is a constant magnetic flux density B perpendicular to the plane of the rails. Neglecting the resistance of the rails and the rod, and the self-inductance of the circuit:

(a) calculate the current induced in the circuit;

(b) calculate the external force required to maintain steady motion of the rod;

(c) calculate the power P_1 supplied to maintain the steady motion of the rod;

(d) compare P_1 with the power P_2 dissipated in the resistor and comment on the result.

F.3 Homopolar generator. A conducting circular disc of radius a=3 m and mass m=10^4 kg rotates about its axis with angular frequency ω=3000 min^{-1} in a uniform field of magnetic flux density B=0.5 T parallel to its axis.

(a) Show that the potential difference V between the axis and the rim of the disc is $\omega a^2 B/2$.

(b) A load resistor of R=$10^{-3}\,\Omega$ is connected suddenly between the rim and the axis of the disc. What is the initial value of the current in the load (neglecting any other resistance in the circuit)? How long would it take for the flywheel to slow to half its initial speed in the absence of mechanical friction?

F.4 Self-inductance of a coax-cable. A co-axial cable is made from concentric cylindrical conductors. The radius of the inner conductor is a and the outer conductor has an inner radius b and an outer radius d. Calculate the self inductance per unit length of the cable. You may assume that $(b-a) \gg a$ and that $(b-a) \gg (d-b)$. Why is this assumption necessary?

F.5 Mutual induction between a small and a large coil. A small coil of N turns and area A, carrying a constant current I, and a circular ring with radius R have a common axis. The small coil moves along the axis so that its distance from the centre of the ring is given by $d = d_0 + a\cos(\omega t)$. Show that the voltage V induced in the ring is given by:

$$V = \frac{3}{2}\,\mu_0 N A I \omega\,\frac{a R^2 d}{(R^2+d^2)^{5/2}}\,\sin(\omega t).$$

F.6 Mutual inductance of two coaxial solenoids Two coaxial, completely overlapping solenoids, each having n turns per unit length and total length l, have radii a and $2a$.

(a) For each individual solenoid carrying a current I, find the magnetic flux density B generated within the solenoid at any point far removed from the ends.

(b) Neglecting end effects, calculate the self-inductance of each coil and the mutual inductance of the coils.

(c) The outer coil has a self-inductance of 40 mH. Calculate the e.m.f. induced in the inner coil when a current in the outer coil collapses at a constant rate of $2\,\mathrm{As}^{-1}$.

F.7 Energy of the magnetic field. Show that the energy stored in an inductor of self-inductance L carrying a current I can be written as $\frac{1}{2}LI^2$. Show that hence the magnetic energy per unit volume associated with the magnetic flux density B inside the coil can be expressed as $\frac{1}{2}B^2/\mu_0$.