First-Year Electromagnetism: Problem Set 1

Hilary Term 2015, Prof LM Herz

A. Electric Fields, Potentials and the Principle of Superposition

A.0 Background. State the definition of the electric field and potential and derive their relationship. Explain how the principle of superposition applies to charge distributions in electrostatics.

A.1 Assembly of point charges in the corners of a square. Charges $+q$, $+2q$, $-5q$ and $+2q$ are placed at the four corners ABCD of a square of side a, taken in cyclic order.

(a) Find the electric field $\mathbf{E}$ and the potential V at the centre of the square and verify that they are related by $\mathbf{E} = -\nabla V$.

(b) What is the potential energy of the charge configuration, i.e. the work done in assembling the configuration, starting with all the charges at infinity?

[Answers: $12q/(4\pi\epsilon_0 a^2)$ towards C; 0; $-q^2(32+\sqrt{2})/8\pi\epsilon_0 a$]

A.2 Electric dipole. Two point charges $\pm q$ are placed at points $(0, 0, \pm d/2)$, defining an electric dipole moment $\mathbf{p} = q\mathbf{d}$.

(a) Using spherical polar coordinates, show that the potential V a large distance $r = |\mathbf{r}|$ from the dipole is given by

$$V = \frac{\mathbf{p} \cdot \mathbf{r}}{4\pi\epsilon_0 r^3}$$

(b) Derive expressions for the electric field vector $\mathbf{E} = (E_r, E_\theta, E_\phi)$ for large r.

(c) Determine the energy W of a dipole placed with its moment $\mathbf{p}$ at an angle α to the direction of an external electric field $\mathbf{E_{ext}}$.

A.3 Assembly of point charges on a line; multipoles. A system of charges consists of one charge $+q_2$ at the origin and two charges $-q_1$ at points (0, 0, $\pm a$).

(a) Using spherical polar coordinates, find the potential $V(r, \theta, \phi)$ created by these charges, taking θ to be the angle between $\mathbf{r}$ and the z-axis.

(b) Expand the potential as a power series in a/r, retaining only terms up to the second order. State which parts of your expression have monopole, dipole and quadrupole character.

(c) For the case of $q_2 = 2q_1$, state the potential and derive expressions for the radial and angular components of the associated electric field.

A.4 Uniformly charged rod. A thin rod of length $2l$ is uniformly charged with charge λ per unit length. By integrating the electric field components originating from small elements of the rod, calculate the total electric field outside the rod for:

(a) any point on the line of the rod (but beyond its ends) as a function of distance z from its mid point.

(b) any point a perpendicular distance x away from the midpoint of the rod.

A.5 Uniformly charged disk. A thin, circular disk has radius b and carries a surface charge density σ. Consider the disk to lie in the x-y-plane with its centre at the origin.

(a) Find the electric field $\mathbf{E}$ for any point P on the z-axis.

(b) What are the values of $\mathbf{E}$ for the limiting cases of $z \ll b$ and $z \gg b$?

(You can solve this problem by calculating the field at P arising from a ring of charge of radius r and width dr, and then integrating from $r = 0$ to $r = b$.)

A.6 Uniformly charged ring. The disk in the previous question is replaced by a thin, uniformly charged ring of radius a carrying charge q.

(a) Determine the points on the axis of the ring for which the magnitude of the electric field $|E_z|$ reaches its maximum value.

(b) Show that an electron placed on the z-axis at a small distance ($z \ll a$) from the centre of the ring will oscillate with frequency

$$\nu = \sqrt{\frac{eq}{16\pi^3\epsilon_0 a^3 m}}.$$

A.7 Uniformly charged hollow sphere. A charge is distributed uniformly with density σ over the surface of a hollow conducting sphere of radius a. Show by direct integration over contributions arising from infinitesimal surface elements of the sphere that the potential at any point P inside it is given by $a\sigma/\epsilon_0$.

(Hint: orienting the z-axis to contain P and using polar coordinates will make your life easier.)